AIDS
TO THE
MATHEMATICS OF HYGIENE

STUDENTS' AIDS SERIES

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R. B. FERGUSON

FIFTH EDITION

BAILLIÈRE, TINDALL & COX

Government Grant.

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THE MATHEMATICS OF HYGIENE

BY

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PREFACE TO THE FIFTH EDITION

In preparing the fifth edition of this book, I have taken the opportunity not only to revise it, but to bring it up to date by the introduction, amongst other things, of a note on the new method of recording barometrical observations, and the estimation of the calorific values of foods—a matter which is attracting a good deal of attention at the present time.

Many of the existing examples have been replaced by others, of a similar form, taken from papers recently set for the Diploma in Public Health by different examining bodies, in the hope they may prove a useful guide to intending candidates as to the nature and extent of the mathematical knowledge required from them.

R. B. F.

Forest Side,
Epping.
October, 1919.

PREFACE TO THE SECOND EDITION

THE issue of a second edition of this little book has afforded me the opportunity of revising it throughout. A certain amount of new matter and some additional examples have been introduced, whilst the chapter on Vital Statistics has been re-written and enlarged. On the other hand, such portions have been deleted as appeared to stray beyond the limits I originally laid down for myself—viz., to deal with the study of Hygiene solely from the *mathematical* side of the subject.

This side has always seemed to me to be treated rather inadequately by even the standard text-books. In these, a formula is inserted in its entirety, without a hint as to how such a result has been arrived at, or by what method it was compiled. Without this knowledge, the only alternative is to commit it to memory—with the risk of forgetting it—before entering for an examination on the subject.

It is also generally taken for granted that the reader is well acquainted with all the methods of chemical and physical calculations; but since these subjects are amongst the earliest of one's scientific studies, it is quite within the range of possibility that these details may have passed somewhat from recollection ; so that, should he be preparing for examination in Public Health, they must be all sought out at infinite trouble from, possibly, a dozen different sources (chemistry, physics, algebra, statics, dynamics, trigonometry, etc.).

It has been my endeavour to collect these formulæ, and to put them together in these pages, chiefly in the form of examples. I have also attempted to show, in as simple a manner as possible, how they may be applied and made use of, in the solution of the various problems so frequently met with during the course of one's study. The book is not meant to be a guide to Hygiene, nor yet to embrace every branch of the subject. It is only what its title claims for it—viz., a guide to those portions of Public Health which require mathematical treatment. As such, it is gratifying to find that it has already been of some assistance; and it is hoped that it will continue to prove useful, especially to those who have not the time, nor yet, perhaps, the inclination, to commence their mathematical studies afresh.

The calculations connected with the practical examination of milk, water, etc., have been omitted, since these are so inseparably connected with the actual laboratory work, that their inclusion here would necessitate the introduction of the whole methods of analysis—a subject already fully dealt with in the many special text-books.

R. B. F.

October, 1903.

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TO THE

MATHEMATICS OF HYGIENE

CHAPTER I

LAWS OF GASES

Definitions :

Gases are bodies whose molecules are in a constant state of motion, in virtue of which they possess the most perfect mobility, and are continually tending to occupy a greater space. They are also called 'elastic fluids.' A minute quantity of a gas may be made to occupy any space, however large, by sufficiently reducing the pressure to which it is exposed.

Vapours are the aeriform state into which liquids—such as water or alcohol—are converted by the application of heat.

A gas obeys the same laws—as to temperature and pressure—as atmospheric air. A vapour only obeys these laws within certain limits. Non-saturated vapours resemble gases, and therefore obey the laws of Boyle and Charles (p. 5). Saturated vapours do not, for if the pressure be too great or the temperature too low, a portion of the vapour will at once pass into the liquid state ; whilst if the pressure be reduced or the temperature raised, they will no longer be saturated. Thus for a given temperature, vapours, when saturated and in contact with their liquid, can neither be compressed nor expanded.

Gases and the Relationship of their Volume, Weight, Pressure, and Temperature.

Volume and Weight :

The weight of a given volume of any gas or vapour can readily be calculated if we know—

(i.) The weight of a given volume of some gas which can be taken as a standard ;

(ii.) The relative density of the gas, compared with this standard.

In Chemistry it is usual to take, as the standard, the weight of a given volume of *hydrogen* at a certain temperature and pressure ; and the weight of any other gas, at the same temperature and pressure, is obtained by multiplying this by the relative density of the gas whose weight is being ascertained.

The standard always adopted is that known as the 'crith,' and is :

1 litre of hydrogen at 0° C. and 760 mm. weighs 0·08958 gramme.

Now, the densities of all **elements** in the gaseous state are identical with their atomic weights ; and since $O=16$, and $S=32$, therefore

1 litre of oxygen at 0° C. and 760 mm. weighs $16 \times 0{\cdot}08958 = 1{\cdot}43$ grammes ;

and 1 litre of sulphur vapour at 0° C. and 760 mm. weighs $32 \times 0{\cdot}08958 = 2{\cdot}86$ grammes.

The density of a **compound** gas is one-half its molecular weight; and since $CO_2 = 44$, and $H_2O = 18$, therefore the density of $CO_2 = 22$, and the density of water-vapour (steam) = 9, compared with that of H as unity.

∴ 1 litre of CO_2 at 0° C. and 760 mm. weighs
$22 \times 0{\cdot}08958 = 1{\cdot}97$ grammes.

For convenience of calculation, the relationship between the weight and volume of a gas may be expressed as follows:

1 litre of H at 0° C. and 760 mm. weighs 0·08958 grm.;

∴ $\frac{1}{0{\cdot}08958}$ litre weighs 1 gramme;

i.e., 11·2 litres of	H	weigh	1 gramme.	
∴ 11·2 ”	O	”	16 grammes;	
and 11·2 ”	S	”	32	”
11·2 ”	CO_2	”	22	”
11·2 ”	steam	”	9	”

That is, *the weight of* 11·2 *litres of any gas at* 0° *C. and* 760 *mm. is its density expressed in grammes.*

In Physics, it is usual to take *air* as the standard, and the density or weight of a gas or vapour is the relation between the weight of a given volume of this gas or vapour, and that of the same volume of air at the same temperature and pressure.

The standard here adopted is:

1 cubic foot of dry air at 32° F. and 760 mm. weighs 566·85 grains.

Since air is 14·47 times heavier than hydrogen, the weight of any gaseous element, or compound, may be found as follows:

Density of oxygen : density of air : : 16 : 14·47 ;
Density of water-vapour : density of air : : 9 : 14·47 ;

and so for any other gas.

[NOTE ON PROPORTION.—Four quantities are said to be proportional when the ratio of the first to the second is equal to the ratio of the third to the fourth. Thus, if a, b, c, and d are proportional—that is, $a : b : : c : d$—then $\frac{a}{b} = \frac{c}{d}$, or $\frac{a}{c} = \frac{b}{d}$, or $ad = bc$.]

Example:

Find the weight of 1 cubic foot of SO_2 at 32° F. and 760 mm.

$$\text{Since } \frac{\text{weight of } SO_2}{\text{weight of air}} = \frac{32}{14{\cdot}47};$$

$$\therefore \frac{1 \text{ cubic foot } SO_2}{566{\cdot}85} = \frac{32}{14{\cdot}47};$$

$$\therefore 1 \text{ cubic foot } SO_2 = 566{\cdot}85 \times \frac{32}{14{\cdot}47} = 1253{\cdot}57 \text{ grains.}$$

Since the coefficient of expansion of all gases is the same as that of air, it does not signify at what temperature or pressure the relative densities are compared, provided they are both at the same temperature and pressure ; *e.g.*, the relative densities of air and aqueous vapour (the pressure being constant) would be the same at 60° F. as at 32° F., provided both are at 60° F. or 32° F. respectively; but the same fraction would not represent the relative density between air at 32° F. and aqueous vapour at 60° F.

Example:

The relative density of air, where hydrogen is the standard, being 14·47, find the relative densities of oxygen, hydrogen, carbon-dioxide, and aqueous vapour, taking air as the standard.*

Density of oxygen : density of air : : 16 : 14·47 ;

$$\text{or, } \frac{\text{density of O}}{\text{density of air}} = \frac{16}{14{\cdot}47}.$$

Take density of air = 1, then

$$\text{density of O} = \frac{16}{14{\cdot}47} = 1{\cdot}1.$$

Density of hydrogen : density of air : : 1 : 14·47 ;

$$\therefore \text{ density of H} = \frac{1}{14{\cdot}47} = 0{\cdot}069.$$

Density of carbon-dioxide : density of air : : 22 : 14·47 ;

$$\therefore \text{ density of } CO_2 = \frac{22}{14{\cdot}47} = 1{\cdot}52.$$

Density of aqueous vapour : density of air : : 9 : 14·47 ;

$$\therefore \text{ density of aqueous vapour} = \frac{9}{14{\cdot}47} = 0{\cdot}622.$$

(In the following examples, W, V, P, and T represent the present—or existing—weight, volume, pressure, and temperature respectively ; while *w*, *v*, *p*, and *t*, represent the new weight, volume, etc., which are to be ascertained.)

Example :

What are the laws of the volume of gases—(*a*) to pressure, (*b*) to temperature ? Illustrate by an example of each.†

(*a*) **Volume and Pressure** :

Boyle's (*or Mariotte's*) *law* states that the temperature being constant, the volume (V) varies inversely as the pressure (P).

That is, V : *v* : : *p* : P

* D.P.H. Exam., Cambridge.
† D.P.H. Exam. (Roy. Coll. Phys. Surg.).

Example:

A gas measures 100 c.c. at 730 mm. Find its volume at 760 mm.

$$V : v :: p : P;$$

$$\therefore 100 : v :: 760 : 730;$$

$$\therefore v = \frac{100 \times 730}{760} = 96 \text{ c.c.}$$

(*b*) **Volume and Temperature:**

Charles's (or Gay Lussac's) law states that all gases expand $\frac{1}{273}$ part of their volume at 0° C. for every increase in temperature of 1° C., or $\frac{1}{491}$ part of their volume at 32° F. for every increase in temperature of 1° F.; *i.e.*, 1 vol. at 0° C. becomes at 10° C. $1 + \frac{10}{273}$, or $\frac{273+10}{273}$ vols., and 1 vol. at 32° F. becomes at 60° F. $1 + \frac{60-32}{491}$, or $\frac{491+(60-32)}{491}$ vols.

Charles's law may therefore be stated as follows: The pressure being constant, the volume of a gas varies directly as the absolute temperature (the absolute temperature for the Centigrade scale being the temperature + 273)—*i.e.*,

$$V : v :: 273 + T : 273 + t;$$

or for the Fahrenheit scale:

$$V : v :: 491 + (T - 32) : 491 + (t - 32).$$

Examples:

(i.) What volume would 1,000 c.c. of a gas at 0° C. expand to at 80° C.?

$$V : v :: 273 + T : 273 + t;$$
$$\therefore 1{,}000 : v :: 273 + 0 : 273 + 80;$$
$$:: 273 : 353;$$
$$\therefore v = \frac{1{,}000 \times 353}{273} = 1{,}293 \text{ c.c.}$$

(ii.) What volume would 1,000 c.c. of a gas at 40° C. expand to at 120° C. ?*

The rise in temperature is 80° C., as in the preceding example; but in this case the expansion takes place in the ratio of 273 + 40 : 273 + 120.

$$V : v :: 273 + T : 273 + t$$
$$\therefore 1{,}000 : v :: 273 + 40 : 273 + 120$$
$$:: 313 : 393;$$
$$\therefore v = \frac{1{,}000 \times 393}{313} = 1255 \text{ c.c.}$$

Pressure and Weight:

It has been seen from Boyle's law that, the temperature remaining the same, the volume of a gas varies inversely as the pressure, or $P \propto \frac{1}{V}$; and as the quantity of gas remains the same, its density must obviously increase as its volume diminishes—that is, density varies inversely as volume, or $D \propto \frac{1}{V}$. It follows, therefore, that density varies directly as pressure, (or $D \propto P$)—that is to say, that for the same temperature, the density of a gas, and therefore its weight, is proportional to its pressure; or,

$$W : w :: P : p.$$

* D.P.H. Exam. (Roy. Coll. Phys. Surg.).

Temperature and Weight:

Since the volume varies directly as the absolute temperature—Charles's law—and the density diminishes in proportion to the increase of volume, therefore the density (and, consequently, the weight) varies inversely as the absolute temperature ; or,

$$W : w :: 273+t : 273+T;$$

or for the Fahrenheit scale :

$$W : w :: 491+(t-32) : 491+(T-32).$$

Pressure and Temperature:

When the volume is constant, the pressure varies directly as the absolute temperature ; or,

$$P : p :: 273+T : 273+t.$$

Example:

Steam at 100° C.—and, therefore, at a pressure of 760 mm.—is removed from contact with water, and passed into the closed chamber of a Disinfector, and heated until the pressure-gauge registers 790 mm. Find the temperature of the steam in the chamber.

The steam, being superheated, is consequently a non-saturated vapour, and as such obeys the laws of gases (p. 1).

$$P : p :: 273+T : 273+t;$$
$$\therefore 760 : 790 :: 273+100 : 273+t;$$
$$\therefore 760\,(273+t)=790\times 373;$$
$$\therefore t=\frac{790\times 373}{760}-273$$
$$=115^\circ\text{ C.}$$

Example (involving variations in temperature, pressure, and volume):

A gas at 7° C. and 760 mm. occupies 15 litres. Find its volume at −8° C. and 720 mm.

(i.) Find volume due to change of pressure:

$$V : v :: p : P;$$
$$\therefore 15 : v :: 720 : 760;$$
$$\therefore v = \frac{15 \times 760}{720} = 15{\cdot}83 \text{ litres.}$$

(ii.) Find volume of this new volume, due to change of temperature:

$$V : v :: 273 + T : 273 + t;$$
$$\therefore 15{\cdot}83 : v :: 273 + 7 : 273 - 8;$$
$$:: 280 : 265;$$
$$\therefore v = \frac{15{\cdot}83 \times 265}{280} = 14{\cdot}98 \text{ litres}$$

Examples of this nature may also be solved by the algebraical process known as *Variation*. It has been seen that where the pressure (P) is constant, the volume (V) varies as the absolute temperature (A), or $V \propto A$; that is, the ratio $\frac{V}{A}$ is constant. If m = this constant ratio, then $\frac{V}{A} = m$, and $V = mA$.

Similarly, where the temperature is constant, V varies inversely as P, or $V \propto \frac{1}{P}$; that is, $V = m\frac{1}{P}$, or $VP = m$.

Where both temperature and pressure vary, V will vary as the product $A \times \frac{1}{P}$; that is $V \propto \frac{A}{P}$, and $V = m\frac{A}{P}$. Therefore, $m = \frac{VP}{A}$.

In the preceding example—

$$V = 15,\ P = 760,\ A = 273 + 7 = 280;$$

$$\therefore\ m = \frac{15 \times 760}{280} = 40{\cdot}7.$$

$$\text{New volume} = m\,\frac{A}{P}.$$

Where $m = 40{\cdot}7$, $A = 273 - 8 = 265$, $P = 720$;

$$\therefore\ V = \frac{40{\cdot}7 \times 265}{720} = 14{\cdot}98 \text{ litres.}$$

DIFFUSION OF GASES.

Example:

What is the law relating to the diffusion of gases?*

If two different gases be separated by a porous diaphragm, an exchange takes place between them, and ultimately the composition of the gas on both sides of the diaphragm is the same; but the rapidity with which this diffusion occurs varies with the gases.

Graham's law states that the force of diffusion varies inversely as the square roots of the densities of the gases.

Thus H and O will diffuse as follows:

$$H : O :: \sqrt{16} : \sqrt{1};$$
$$:: 4 : 1.$$

Or, for every one part of O which has passed into the H, four parts of H have passed into the O.

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* D.P.H. Exam. (Roy. Coll. Phys. Surg.).

CHAPTER II

HYGROMETRY

DRY AIR.

To find the Weight of a Given Volume of Dry Air at a Given Temperature and Pressure.

FROM what has been shown in the previous chapter, a general formula may now be constructed which will embrace all the variations in volume, pressure, and temperature to which any weight of gas may be subjected, as follows:

1 cubic foot of dry air at 0° C. and 760 mm. weighs 566·85 grains;

and since weight varies directly as volume,

$$\therefore w : W \quad :: v : V;$$
$$\text{or, } w : 566{\cdot}85 :: v : 1 \ldots\ldots \text{(i.)};$$

and since weight varies directly as pressure,

$$\therefore w : W \quad :: p : P;$$
$$\text{or, } w : 566{\cdot}85 :: p : 760 \ldots\ldots \text{(ii.)};$$

and since weight varies inversely as absolute temperature,

$$\therefore w : W \quad :: 273+T : 273+t;$$
$$\text{or, } w : 566{\cdot}85 :: 273+0 : 273+t;$$
$$:: 273 : 273+t \ldots\ldots \text{(iii.)}$$

If these three proportions—(i.), (ii.), and (iii.)—be combined, we have:

$$w : 566{\cdot}85 :: v : 1$$
$$:: p : 760;$$
$$:: 273 : 273+t;$$

$$\therefore \frac{w}{566{\cdot}85}=\frac{v}{1}\times\frac{p}{760}\times\frac{273}{273+t};$$

$$\therefore w=566{\cdot}85\times v\times\frac{p}{760}\times\frac{273}{273+t} \text{ grains};$$

or for the Fahrenheit scale:

$$w=566{\cdot}85\times v\times\frac{p}{760}\times\frac{491}{491+(t-32)} \text{ grains};$$

(where w=weight in grains, and v=vol. in cubic feet).

Example:

Find the weight of 1 cubic foot of dry air at 60° F. and 735 mm.

$$w=566{\cdot}85\times 1\times\frac{735}{760}\times\frac{491}{491+(60-32)}=518 \text{ grains.}$$

AQUEOUS VAPOUR.

To find the Weight of a Given Volume of Aqueous Vapour at a Given Temperature and Pressure.

To find the weight of a vapour, the weight of the same volume of dry air *at the same temperature and pressure* must be sought, and this is then to be multiplied by the relative density of the vapour.

Example:

Find the weight of a cubic foot of aqueous vapour at 60° F.

From a table of vapour-tensions (p. 185), it is found that the maximum pressure which aqueous vapour can exert at 60° F.=13·167 mm.

Therefore, the weight of a cubic foot of dry air at 60° F. and 13·167 mm. must first be found, as follows (p. 11):

$$w = \frac{566 \cdot 85 \times 13 \cdot 167 \times 491}{760 \times [491 + (60 - 32)]} = 9 \cdot 29 \text{ grains};$$

then this result must be multiplied by the relative density of aqueous vapour compared with air — viz., by 0·622 (p. 5).

Therefore, 1 cubic foot of aqueous vapour at 60° F. weighs $9 \cdot 29 \times 0 \cdot 622 = 5 \cdot 77$ grains.

Example:

What weight of aqueous vapour is contained in a cubic foot of air which is saturated at a temperature of 60° F.?

Dalton's law states that 'the tension, and consequently the quantity, of vapour which saturates a given space, are the same for the same temperature, whether this space contains a gas or is a vacuum.' Thus, the formation of vapour does not depend upon the presence of air, or upon its density. If water be introduced into two similar vessels, the one containing air and the other a vacuum, the quantity of vapour formed will be the same in each case. Therefore, the weight of aqueous vapour in a cubic foot of air is the same as if the space had been empty of air; so the question resolves itself into, What is the weight of a cubic foot of aqueous vapour at 60° F.?

[Ans.: 5·77 grains.]

SATURATED AIR.

To find the Weight of a Given Volume of Saturated Air at a Given Temperature and Pressure.

The mass of air may be divided into two parts—viz., a volume of dry air, and a volume of aqueous vapour; and the sum of the weights of these two volumes is the weight required—*i.e.*, the actual pressure of the mixture is the sum of the pressures due to the gas and vapour considered separately.

Let P = the pressure of the moist air;
and p = the elastic force of the vapour which saturates it.

Then the *air* alone in the mixture only supports a pressure of $P - p$.

Example:

Find the weight of 1 cubic foot of saturated air, at 60° F. and ordinary atmospheric pressure.

(i.) Find the weight of 1 cubic foot of dry air at 60° F. and pressure $P - p$ (p. 11).

$P = 760$, and $p = 13·167$ (p. 185).

$$\therefore w = 566·85 \times \frac{760 - 13·167}{760} \times \frac{491}{491 + (60 - 32)}$$

$$= 526·97 \text{ grains.}$$

(ii.) Find the weight of 1 cubic foot of aqueous vapour at 60° F. and pressure p (p. 12).

$$w = \frac{566·85 \times 13·167 \times 491}{760 \times [491 + (60 - 32)]} \times 0·622 = 5·77 \text{ grains;}$$

$\therefore$ Weight of 1 cubic foot of saturated air at 60° F. and 760 mm. is:

$$526·97 + 5·77 = 532·74 \text{ grains.}$$

Note.—It is seen that 1 cubic foot of *saturated* air at 60° F. and ordinary atmospheric pressure weighs 532·74 grains, whereas 1 cubic foot of *dry* air at 60° F. and the same pressure weighs $\frac{566·85 \times 491}{491 + (60 - 32)} = 536·27$ grains.

That is to say, the saturated air weighs less than an equal volume of dry air. The explanation of this is as follows:

Dry air expands on taking up moisture, and when 1 cubic foot of dry air at 60° F., weighing 536·27 grains, takes up 1 cubic foot of aqueous vapour (at the same temperature and pressure) weighing 5·77 grains, the weight of the resulting moist air will be $536{\cdot}27 + 5{\cdot}77 = 542{\cdot}04$ grains, but the volume of the mixture will be, not 1 cubic foot, but somewhat more than 1 cubic foot. The moist air is under a pressure of 760 mm., and it has been already seen that the elastic force of the vapour which saturates the air is 13·167 mm. Therefore, the air in the mixture supports a pressure of $760 - 13{\cdot}167 = 746{\cdot}833$ mm. only, and, as volume (v) is inversely as pressure, then

$$v : 1 \text{ cubic foot} :: 760 : 746{\cdot}833;$$

$$\therefore v = \frac{760}{746{\cdot}833} = 1{\cdot}0176 \text{ cubic feet.}$$

That is, the resulting mixture of the dry air and the vapour produces 1·0176 cubic feet of saturated air, weighing 542·04 grains, and 1 cubic foot of this same moist air would only weigh

$$\frac{542{\cdot}04}{1{\cdot}0176} = 532{\cdot}7 \text{ grains.}$$

To find the Dew-point.

For any given temperature, air will only hold a certain quantity of aqueous vapour ; the higher the temperature of the air, the greater will be the amount of vapour which it can contain ; when the air contains its greatest possible amount, it is said to be 'saturated,' and the temperature at which saturation occurs is called the 'dew-point.'

The dew-point may be obtained **directly** by means of a Hygrometer (Daniell's, Regnault's, or Dine's), or **indirectly** by the dry-and-wet bulb Hygrometer.

In the case of the former, the dew-point is the temperature at which the thermometer in the blackened bulb stands, at the moment when deposition of moisture on the bulb occurs. Whereas, in the case of the dry-and-wet-bulb, the wet-bulb does *not* indicate the dew-point, which, therefore, cannot be ascertained by simple inspection of the thermometer. If the air be saturated no evaporation is possible, and the two thermometers will read alike ; if not saturated, the wet-bulb will read lower than the dry-bulb, but not so low as the dew-point ; in fact, the temperature of the wet-bulb is always above the dew-point. When the dry bulb stands at 53° F. the dew-point is as much below the wet-bulb as the wet-bulb is below the dry-bulb. Above this temperature, the wet-bulb approaches nearer the dew-point, and the reverse is the case below that temperature.

Glaisher has empirically compiled some tables, whereby the difference between the dew-point and the wet-bulb bears a constant ratio to the difference between the wet-bulb and the dry-bulb ; so that, if the reading of the dry-bulb be given, the dew-point can be calculated.

According to him, the temperature of the dew-point is obtained by multiplying the difference between the wet- and dry-bulb temperatures by a constant factor ('Glaisher's factor'), and subtracting the product thus obtained from the dry-bulb temperature, thus :

$$\text{Dew-point} = T_d - [(T_d - T_w) \times F].$$

Where T_d = temperature of dry-bulb (Fahrenheit).
T_w = temperature of wet-bulb ,,
F = factor (found opposite the dry-bulb temperature in the table).

If $T_w = 60°$ F. and $T_d = 54°$ F., then (from table)

$$F = 1{\cdot}88 \text{ and}$$

$$\text{dew-point} = 60 - [(60 - 54) \times 1{\cdot}88] = 48{\cdot}72° \text{ F.}$$

From this formula Glaisher calculated his tables, which give the dew-point on inspection.

The dew-point may also be found by *Apjohn's formula.* For this purpose a table of vapour-tensions (p. 185) is required.

His formula is as follows :

$$F = f - \left(\frac{d}{87} \times \frac{h}{30}\right)$$ for temperatures above 32° F., and

$$F = f - \left(\frac{d}{96} \times \frac{h}{30}\right)$$ for temperatures below 32° F.

Where F = tension of vapour at dew-point,

f = tension of vapour at temperature of wet-bulb.

d = difference (Fahrenheit) between the wet-and-dry-bulb thermometers.

h = height of barometer (in inches).

Near the sea-level, the fraction $\frac{h}{30}$ differs very little from unity, and may be neglected; so the formula may be simplified thus :

$$F = f - \frac{d}{87}, \text{ or } F = f - \frac{d}{96}.$$

The constant 87 (or 96) represents the specific heat of air and vapour.

Having found F, the table of vapour-tensions must be again referred to, and the temperature opposite the tension F will be the dew-point.

To recapitulate the more important points—

The dew-point may be found in two different ways:

(i.) By direct observation of thermometer, in Daniell's, Regnault's, or Dine's Hygrometer.

(ii.) Indirectly, by the dry-and-wet-bulb Hygrometer—
(*a*) By means of Glaisher's tables.
(*b*) By means of Apjohn's formula.

It must be noted that Glaisher's formula gives the dew-point directly, no table of vapour-tensions being required; whereas Apjohn's formula requires the use of a vapour-tension table, the result being *not* the dew-point, but the vapour-tension at dew-point, from which the dew-point can be ascertained. And attention is again drawn to the fact, that the wet-bulb thermometer does not give the dew-point (except when the air is saturated), and therefore the wet-and-dry-bulb Hygrometer must not be confounded with any of the *direct* Hygrometers—*e.g.*, Daniell's, where the thermometer *does* give the dew-point.

To find the Humidity and the Drying Power of the Air.

Example:

The temperature of a room is 60° F., and the dew-point is 50° F. Find the degree of humidity of the room, and the drying power of the air at the time of observation.

$$\text{R. H.} = \frac{\text{weight of water actually present in given vol. of air}}{\text{weight of water which would \textbf{saturate} the same vol.}}$$

The water-vapour actually present is enough to saturate the air at 50° F., but not enough to do so at 60° F. This gives the actual tension of the water-vapour present in the air at 60° F.; for it must be the same as the *maximum tension* at 50° F. In other words, the actual vapour pressure in any portion of air is equal to the maximum vapour pressure at dew-point.

Let P_{50} = maximum tension at 50° F., and
P_{60} = „ „ „ 60° F.

Then, the weight of vapour actually present (*e.g.*, in a cubic foot) will be (p. 12):

$$\frac{566\cdot85 \times P_{50} \times 491}{760 \times [491 + (60 - 32)]} \times 0\cdot622 = P_{50} \times 0\cdot439 \text{ grains,}$$

and the weight of vapour which would saturate the same volume will be:

$$\frac{566\cdot85 \times P_{60} \times 491}{760 \times [491 + (60 - 32)]} \times 0\cdot622 = P_{60} \times 0\cdot439 \text{ grains;}$$

$$\therefore \text{R.H.} = \frac{P_{50} \times 0\cdot439}{P_{60} \times 0\cdot439} = \frac{P_{50}}{P_{60}} = \frac{\text{maximum tension at } 50^\circ \text{ F.}}{\text{maximum tension at } 60^\circ \text{ F.}}$$

$$= \frac{\text{maximum tension at dew-point}}{\text{maximum tension at existing temperature}},$$

which, from the table (p. 185), will be found to be $\frac{9\cdot165}{13\cdot167} = 0\cdot696$, or 69·6 (approximately 70) per cent.

So the **relative humidity** of the air can always be found from a table of maximum vapour-tensions, if only the dew-point and the temperature of the air at the time of observation be known.

The **drying power** of the air means the additional weight of vapour necessary to cause saturation.

If relative humidity = 70 per cent., then amount of vapour actually present in a given volume of air is 70 per cent. of saturation, and the difference (or 30 per cent.) will represent the drying power.

Thus, if W = weight of vapour required to *saturate* a cubic foot of air at 60° F., and relative humidity = 70 per cent., then the amount of vapour actually present will be $\frac{70W}{100}$, and the drying power of the same air will be $\frac{30W}{100}$. And since the weight of vapour in a saturated cubic foot of air at 60° F. is 5·77 grains (p. 13), therefore the amount of vapour actually present is $5{\cdot}77 \times \frac{70}{100} = 4{\cdot}03$ grains, and the drying power is the difference between these—viz., $5{\cdot}77 - 4{\cdot}03 = 1{\cdot}7$ grains.

To find the Weight of a Given Volume of Moist (non-saturated) Air at a Given Temperature and Pressure, the Hygrometric State or Relative Humidity being given.

Example:

Find the weight of 1 cubic foot of moist air at 60° F. and ordinary atmospheric pressure, the relative humidity of the air being 60 per cent.

It has been seen (p. 14) that 1 cubic foot of moist air at 60° F. is nothing more than a mixture of (i.) 1 cubic foot of dry air at 60° F., under the existing barometric pressure *minus* the tension of the vapour present, and (ii.) 1 cubic foot of aqueous vapour at 60° F., the tension of which must be found from the hygrometric state, or relative humidity.

It has also been seen (p. 19) that:

$$\text{Relative humidity} = \frac{\text{max. tension at dew-point}}{\text{max. tension at existing temp.}}.$$

The existing temperature in this example is 60° F., and from the table (p. 185) it is found that the maximum tension of vapour at 60° F. = 13·167 mm.

$$\therefore \text{Relative humidity} = \frac{\text{max. tension at dew-point}}{13{\cdot}167};$$

$\therefore$ max. tension at dew-point = rel. hum. × 13·167,
and relative humidity = 60 per cent.;

$$\therefore \text{max. tension at dew-point} = \frac{60 \times 13{\cdot}167}{100} = 7{\cdot}9 \text{ mm.}$$

Now, the actual vapour-pressure in any portion of air is equal to the maximum vapour-pressure at dew-point. Therefore:

Actual vapour-pressure in air under observation

$$= 7{\cdot}9 \text{ mm.}$$

The question, then, resolves itself into:

(i.) Find weight of 1 cubic foot of dry air at 60° F. and 760 − 7·9 = 752·1 mm. (p. 11).

$$w = 566{\cdot}85 \times \frac{752{\cdot}1}{760} \times \frac{491}{491 + (60 - 32)} = 530{\cdot}69 \text{ grains.}$$

(ii.) Find weight of 1 cubic foot of aqueous vapour at 60° F. and 7·9 mm. pressure (p. 12).

$$w = 566{\cdot}85 \times \frac{7{\cdot}9}{760} \times \frac{491}{491 + (60 - 32)} \times 0{\cdot}622$$
$$= 3{\cdot}46 \text{ grains.}$$

Therefore, the weight of 1 cubic foot of air (containing 60 per cent. of moisture) at 60° F. =

$$530{\cdot}69 + 3{\cdot}46 = 534{\cdot}15 \text{ grains.}$$

Incidentally, it may be remarked that the temperature at which the aqueous vapour would exert a maximum pressure of 7·9 mm. will be the dew-point, which from the table (p. 185) will be seen to be about 46° F.

To Graduate a Rain-Gauge.

Carefully measure the area of the top of the gauge or receiving surface. Suppose this to be 50 square inches. If, now, this area be covered with water to the height of 1 inch, the quantity of water will be 50 cubic inches. So, to graduate the gauge, 50 cubic inches of water must be put into the glass measure, and a mark placed at the level of the top of the fluid; this mark will represent 1 inch rainfall. The space below may be divided into numerous equal parts, each representing equal fractions of an inch.

The 50 cubic inches of water may be obtained by measurement, as follows:

At 4° C. (or 39·2° F.), or the maximum density point of water, 1,000 fluid ounces = 1 cubic foot = 1,728 cubic inches;

$$\therefore 50 \text{ cubic inches} = \frac{1{,}000 \times 50}{1{,}728} = 28{\cdot}9 \text{ fluid ounces.}$$

If the area of the receiving surface had been 100 square inches instead of 50, then 100 cubic inches of water would represent 1 inch of rainfall. The receiving surface may thus be made of any size; or, on the other hand, any quantity of water may be made to represent any fraction of an inch—*e.g.*, what should be the diameter of the receiving surface so that 1 fluid ounce represents $\frac{1}{8}$ inch?

Let d=diameter in inches,

Then area of receiving surface $= \frac{\pi d^2}{4}$ (p. 178),

and quantity of water standing $\frac{1}{8}$ inch high on receiving surface will be $\frac{\pi d^2}{4} \times \frac{1}{8} = \frac{\pi d^2}{32}$ cubic inches, and this is equal to 1 fluid ounce.

$$\therefore \frac{\pi d^2}{32} = 1 \text{ fluid ounce} = 1{\cdot}728 \text{ cubic inches};$$

$$\therefore d^2 = \frac{32 \times 1{\cdot}728}{\pi} = \frac{32 \times 1{\cdot}728}{3{\cdot}1416} = 17{\cdot}6;$$

$$\therefore d = \sqrt{17{\cdot}6} = 4{\cdot}19 \text{ inches.}$$

THE BAROMETER AND THE THERMOMETER.

Conversion of Thermometer Scales.

Freezing-point = 32° F., 0° C., and 0° R.
Boiling-point = 212° F., 100° C., and 80° R.

Therefore:

$$F - 32 : C : R :: 212 - 32 : 100 : 80;$$
$$:: 180 : 100 : 80;$$
$$:: 9 : 5 : 4.$$

Therefore:

$$\mathbf{\frac{F-32}{9} = \frac{C}{5} = \frac{R}{4}.}$$

Conversion of the Reading of One Scale into that of Another.

Fahrenheit—Centigrade:

$$\frac{F-32}{9} = \frac{C}{5};$$

$$\therefore F = \frac{9}{5} C + 32,$$

$$\text{and } C = \frac{5}{9}(F - 32).$$

Fahrenheit—Réaumur:

$$\frac{F-32}{9} = \frac{R}{4};$$

$$\therefore\ F = \frac{9}{4} R + 32,$$

$$\text{and } R = \frac{4}{9}(F - 32).$$

Centigrade—Réaumur:

$$\frac{C}{5} = \frac{R}{4};$$

$$\therefore\ C = \frac{5}{4} R, \text{ and } R = \frac{4}{5} C.$$

Examples:

1. Find the equivalent of 98·4° F. on the other scales

$$C = \frac{5\ (F - 32)}{9} = \frac{5\ (98{\cdot}4 - 32)}{9} = \frac{5 \times 66{\cdot}4}{9} = 36{\cdot}9;$$

$$R = \frac{4\ (F - 32)}{9} = \frac{4(98{\cdot}4 - 32)}{9} = \frac{4 \times 66{\cdot}4}{9} = 29{\cdot}5;$$

$$\therefore\ 98{\cdot}4^\circ \text{ F.} = 36{\cdot}9^\circ \text{ C.} = 29{\cdot}5^\circ \text{ R.}$$

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2. Find the equivalent of 10° C. on the other scales

$$F = \frac{9}{5} C + 32 = \frac{9 \times 10}{5} + 32 = 18 + 32 = 50;$$

$$R = \frac{4}{5} C = \frac{4}{5} \times 10 = 8;$$

$$\therefore\ 10^\circ \text{ C.} = 50^\circ \text{ F.} = 8^\circ \text{ R.}$$

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3. At what temperature is the number on the Centigrade and Fahrenheit thermometers the same?

Let x = the temperature on F. scale,

then $\frac{5(x - 32)}{9}$ = temperature on C. scale;

and since the temperatures are the same,

$$\therefore\ x = \frac{5(x - 32)}{9}$$

$$9x = 5x - 160$$
$$9x - 5x = -160$$
$$4x = -160$$
$$x = -40^\circ.$$

(It may be noted that this temperature is that of the freezing-point of mercury.)

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4. What is the temperature when the number of degrees on the Centigrade scale is as much below zero as that on Fahrenheit's is above zero?

Let F = temperature on F scale,
C = " " C scale.

And since the temperatures are the same, but on opposite sides of zero—

$$F = -C \;.\;.\;.\;.\;.\; \text{(i.)}$$
$$\text{and } C = \frac{5}{9}(F - 32) \;.\;.\;.\;.\;.\; \text{(ii.)},$$

substitute $-C$ for F in (ii.);

$$\therefore C = \frac{5}{9}(-C - 32)$$
$$9\,C = 5\,(-C - 32)$$
$$= -5\,C - 160;$$
$$\therefore 9\,C + 5\,C = -160$$
$$14\,C = -160$$
$$C = -\frac{160}{14} = -11{\cdot}4^\circ,$$

and $\therefore F = 11{\cdot}4^\circ$.

Conversion of Barometer Scales.

In this country the barometric pressure is frequently stated in inches instead of millimetres. The ordinary atmospheric pressure is 760 mm. = 29·92 inches; and one may be converted into the other by a simple rule of three—*e.g.*, find the value of 735 mm. in inches:

$$760 \text{ mm.} : 735 \text{ mm.} :: 29{\cdot}92 \text{ inches} : x \text{ inches.}$$

$$x = \frac{29{\cdot}92 \times 735}{760} = 28{\cdot}9 \text{ inches.}$$

The Centimetre-Gramme-Second System.

Since May 1, 1914, the metric system has been adopted for meteorological measurements.

In this system the *centimetre* is the unit of length, the *gramme* the unit of mass, and the *second* the unit of time. This system of units is known as the *centimetre-gramme-second* (or C.G.S.) system.

The unit of velocity = 1 centimetre per second.

The unit of force (or the force which produces an acceleration of 1 unit of velocity per second in a mass of 1 gramme) is known as a *dyne.*

The unit of pressure = a dyne per square centimetre.

1,000,000 dynes per square centimetre = a *megadyne.*

As a dyne per square centimetre is an exceedingly small unit, the megadyne per square centimetre has been adopted as the practical unit of atmospheric pressure in the C.G.S. system, and is known as a *bar;* and barometric readings are now given in terms of *centibars* ($\frac{1}{100}$ of a bar) and *millibars* ($\frac{1}{1000}$ of a bar).

The megadyne per square centimetre is equivalent to a pressure of 750·1 mm., or 29·53 inches of mercury;

$$\therefore 29{\cdot}53 \text{ inches} = 1 \text{ megadyne per square centimetre} = 1 \text{ bar} = 1{,}000 \text{ millibars.}$$

Normal atmospheric pressure (760 mm., or 29·92 inches) would thus be $\frac{29{\cdot}92 \times 1{,}000}{29{\cdot}53} = 1{,}013{\cdot}2$ millibars.

Relationship between the Different Varieties of Barometers.

GLYCERINE BAROMETER.

Example:

If the height of a mercurial barometer be 30 inches, calculate what would be the height of a barometer made with glycerine. (Specific gravity of mercury = 13·6, and glycerine = 1·28.)*

Since the height of the column varies inversely as the density,

$$\therefore \begin{Bmatrix}\text{height of}\\ \text{mercury}\end{Bmatrix} : \begin{Bmatrix}\text{height of}\\ \text{glycerine}\end{Bmatrix} :: \begin{Bmatrix}\text{sp. gr. of}\\ \text{glycerine}\end{Bmatrix} : \begin{Bmatrix}\text{sp. gr. of}\\ \text{mercury}\end{Bmatrix};$$

$$\therefore 30 : x :: 1{\cdot}28 : 13{\cdot}6;$$

$$\therefore x = \frac{30 \times 13{\cdot}6}{1{\cdot}28} = 319 \text{ inches} = 26{\cdot}6 \text{ feet.}$$

WATER BAROMETER.

Example:

From what depth is it theoretically possible—with the barometer standing at 30 inches—to raise water with a simple lift-pump?*

The water rises in the pipe till the pressure of the liquid column counterbalances the atmospheric pressure on the water of the reservoir. The pipe is thus, practically, a water barometer. The question, therefore, resolves itself into the following: What is the height of a water barometer, when a mercurial one stands at 30 inches?

Since height varies inversely as density, therefore—

* D.P.H. Exam. (Roy. Coll. Phys. Surg.).

$$\left\{\begin{matrix}\text{height of}\\ \text{mercury}\end{matrix}\right\} : \left\{\begin{matrix}\text{height of}\\ \text{water}\end{matrix}\right\} :: \left\{\begin{matrix}\text{sp. gr.}\\ \text{of water}\end{matrix}\right\} : \left\{\begin{matrix}\text{sp. gr. of}\\ \text{mercury}\end{matrix}\right\}.$$

$$30 : x :: 1 : 13{\cdot}6$$

$$x = 13{\cdot}6 \times 30$$

$$= 408 \text{ inches}$$

$$= 34 \text{ feet.}$$

(In practice, water cannot be raised with a lift-pump more than about 28 feet, owing to the impossibility of obtaining a perfect vacuum.)

Example :

The lower valve of a pump is 30 feet 4 inches above the surface of the water to be raised; find the height of the barometer when the pump ceases to work.

[Ans.: 26·76 inches.]

Alcohol Barometer.

Example:

A mercurial barometer falls 1 inch; what is the corresponding fall in an alcohol barometer?

If the mercury stands at 30 inches, then the height of the alcohol barometer can be found as follows:

$$\left\{\begin{matrix}\text{height of}\\ \text{mercury}\end{matrix}\right\} : \left\{\begin{matrix}\text{height of}\\ \text{alcohol}\end{matrix}\right\} :: \left\{\begin{matrix}\text{sp. gr. of}\\ \text{alcohol}\end{matrix}\right\} : \left\{\begin{matrix}\text{sp. gr. of}\\ \text{mercury}\end{matrix}\right\};$$

$$\therefore 30 : x :: 0{\cdot}8 : 13{\cdot}6;$$

$$x = \frac{30 \times 13{\cdot}6}{0{\cdot}8} = 510 \text{ inches.}$$

That is, 30 inches mercury = 510 inches alcohol;

$$\therefore 1 \text{ inch mercury} = \frac{510}{30} = 17 \text{ inches alcohol.}$$

The alcohol barometer will therefore fall 17 inches.

Correction of Barometric Height for Altitude.

As a rule, the barometer falls 1 inch in ascending 900 feet. If x = number of feet above sea-level of station where observation is taken, then, in ascending that height, the barometer will fall $\frac{1}{900}=0\cdot001$ inch for every foot, or $0\cdot001x$ inch in x feet.

In correcting for altitude, therefore, $0\cdot001x$ inch must be added to the reading taken. That is,

$$\left.\begin{array}{c}\text{Barometric reading}\\ \text{at sea-level}\end{array}\right\}=\text{observed height}+0\cdot001x\text{ inch.}$$

For strict accuracy, the difference in the temperature of the air at the elevated station, and at sea-level, should be taken into account, but for ordinary observations the temperature is assumed to be that of the external air at the station where the barometer is placed.

To Ascertain the Altitude by the Barometer.

If x=barometric height (in inches) at lower station, and y=barometric height (in inches) at upper station, then $(x-y)$ inches = difference between the two readings—*i.e.* $(x-y)$ inch represents the fall.

Since barometer falls 1 inch for every 900 feet ascended, it will fall $(x-y)$ inches in 900 $(x-y)$ feet.

That is, there is a difference of 900 $(x-y)$ feet between the two stations.

To find the altitude, therefore, take barometric readings at both stations, and multiply their difference (in inches) by 900. The product will be the altitude (in feet).

Example:

Barometer at lower station = - - 29·92 inches.
Barometer at upper station = - - 29·21 „

Difference = 0·71 „

∴ Altitude of upper station $=900\times0\cdot71=639$ feet.

Correction of Barometric Height for Temperature.

It is usual to reduce all barometric readings to the freezing-point, in order to render them comparable in different places and at different times.

Let a = coefficient of absolute expansion of mercury,
= 0·0001 for every 1° F.
Let h = height at 32° F., and
h_t = height at t° F.
If volume of mercury at 32° F. = 1, then
volume at t° F. = $1 + a(t - 32)$.

And since height of column varies inversely as the density, and density inversely as the volume; therefore the height must vary directly as the volume, and therefore

$$\frac{h}{h_t} = \frac{1}{1 + a(t - 32)};$$

$$\therefore h = \frac{h_t}{1 + a(t - 32)}.$$

This equation may be further simplified as follows:

Multiply both numerator and denominator by $[1 - a(t - 32)]$; thus:

$$h = \frac{h_t[1 - a(t - 32)]}{[1 + a(t - 32)]\,[1 - a(t - 32)]}$$

$$= \frac{h_t[1 - a(t - 32)]}{1 - [a(t - 32)]^2}.$$

Now a^2 is such an infinitely small number that the expression $[a(t - 32)]^2$ may be neglected without in any appreciable degree affecting the result; the equation therefore becomes:

$$h = h_t\,[1 - a(t - 32)];$$

but $a = 0·0001$,

$$\therefore h = h_t\,[1 - 0·0001\,(t - 32)].$$

[This correction is simply for the expansion of the mercury due to heat, and does not correct for the expansion of the brass-scale.]

Example:

A mercurial barometer reads 30·1 inches at 200 feet above sea-level, and the attached thermometer 65° F. Make the necessary corrections for the correct reading of the barometer at sea-level and 32° F. (Coefficient of expansion of mercury 0·0001 per degree Fahrenheit.)*

Since the barometer falls 1 inch in ascending 900 feet, it will fall $\frac{200}{900} = 0{\cdot}22$ inch in ascending 200 feet; 0·22 inch must, therefore, be added to the observed height in order to correct for altitude. Thus

$$30{\cdot}1 + 0{\cdot}22 = 30{\cdot}32 \text{ inches.}$$

Again,

$$\begin{aligned} h &= h_t\,[1 - 0{\cdot}0001\,(t - 32)] \\ &= 30{\cdot}32\,[1 - 0{\cdot}0001\,(65 - 32)] \\ &= 30{\cdot}32\,[1 - (0{\cdot}0001 \times 33)] \\ &= 30{\cdot}32\,[1 - 0{\cdot}0033] \\ &= 30{\cdot}32 \times 0{\cdot}9967 \\ &= 30{\cdot}22. \end{aligned}$$

Corrected reading = 30·22° F.

THE VERNIER.

Example:

State the principle on which the vernier is constructed.†

The principle of the vernier is as follows:

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* D.P.H. Exam. (Roy. Coll. Phys. Surg.).
† D.P.H. Exam., Cambridge.

A given length of the brass scale is taken, containing n divisions; the same length of the vernier is taken and divided into $n+1$ divisions—that is, the given length contains n divisions on the fixed scale, and $n+1$ divisions on the vernier.

If S = length of one division on the scale,
and V = length of one division on the vernier,

$$\text{then } (n+1)V = nS;$$

$$\therefore V = \frac{n}{n+1}S.$$

If each scale division $= \frac{1}{x}$ inch, then $S = \frac{1}{x}$;

$$\text{and } V = \frac{n}{n+1} \times \frac{1}{x} = \frac{n}{x(n+1)} \text{ inch,}$$

$$\text{and } S - V = \frac{1}{x} - \frac{n}{x(n+1)} = \frac{1}{x(n+1)} \text{ inch.}$$

We thus obtain the following:

Length of scale-division $= \frac{1}{x}$ inch,

" " vernier " $= \frac{n}{x(n+1)}$ inch, and

difference between S and V division $= \frac{1}{x(n+1)}$ inch.

A barometer scale is usually divided into inches, tenths of inches, and twentieths of inches—that is, the distance between the smallest divisions is $\frac{1}{20}$ inch, or $\frac{1}{x} = \frac{1}{20}$.

In graduating the vernier, it is usual to take, for the length, 24 scale-divisions, *i.e.*, $n = 24$; then an equal length of the vernier is divided into $n+1 = 25$ divisions.

A vernier division will thus be less than a scale-division by $\frac{1}{x(n+1)} = \frac{1}{20 \times 25} = 0{\cdot}002$ inch, Therefore, in reading a barometer, the number of divisions read off on the vernier must be multiplied by 0·002, and the product added to the height already observed on the brass scale.

The usual construction of the vernier has been here taken; sometimes, however, n divisions on the scale are divided into $n-1$ divisions on the vernier; in other words, a certain length containing n divisions on the scale may be divided into $n \pm 1$ divisions on the vernier.

CHAPTER III

HYDROSTATICS AND HEAT

SPECIFIC HEAT.

DEFINE the term 'specific heat.' How is the specific heat of bodies determined?*

The **specific heat** (S.H.) of a body = the quantity of heat which it absorbs when its temperature rises through a given range of temperature, compared with the quantity of heat which would be absorbed under the same circumstances by the same weight of water, the specific heat of water being taken as 1.

A **thermal unit** (T.U.) = the amount of heat required to raise one unit of weight (*e.g.*, 1 lb.) of water through one unit of temperature (*e.g.*, 1° C.).

If S.H. = 1, then 1 lb. raised 1° C. = 1 T.U.
If S.H. = 1, „ p lbs. „ 1° C. = p T.U.
If S.H. = 1, „ p lbs. „ t° C. = $p \times t$ T.U.
If S.H. = c, „ p lbs. „ t° C. = $p \times t \times c$ T.U.

That is, the quantity of heat absorbed when a body is heated through a certain range of temperature = weight × temperature × specific heat. Similarly, when the body

* D.P.H. Exam. (Roy. Coll. Phys. Surg.).

cools down again to its original temperature, it parts with the same number of thermal units.

The specific heat of bodies may be determined by (i.) the method of mixtures, or (ii.) the method of melting ice, both of which are sufficiently explained by the two following examples:

(i.) Method of Mixtures.

Example:

A piece of iron, weighing 20 ounces at a temperature of 98° C., is immersed in 60 ounces of water at a temperature of 17° C. After the temperatures have become uniform, that of the cooling-water is found to be 20° C. What is the specific heat of the iron?*

Let c=specific heat of iron.

The heated iron, weighing 20 ounces, has cooled from 98° C. to 20° C.—that is, through 78 degrees—and has, therefore, lost $20 \times 78 \times c$ thermal units. The water, whose weight is 60 ounces, and whose specific heat is 1, has been raised from 17° C. to 20° C.—*i.e.*, through 3 degrees—and has, therefore, absorbed $60 \times 3 \times 1$ thermal units.

The number of thermal units lost by the iron must be the same as the number absorbed by the water.

That is (heat lost)=(heat gained);

$$\therefore 20 \times 78 \times c = 60 \times 3 \times 1;$$

$$\therefore 1560c = 180$$

$$\therefore c = \frac{180}{1560} = 0{\cdot}1153.$$

Therefore specific heat of iron=0·1153, which means that the same quantity of heat which would raise 1 pound

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* D.P.H. Exam. (Roy. Coll. Phys. Surg.).

of iron through 1° C., would only raise 0·1153 pound of water through 1° C.

(ii.) **Method of Melting Ice.**

Example:

A hole is made in a block of ice at 0° C., and an iron ball at 100° C., and weighing 200 grammes, is dropped in. When the ice ceases to melt the water formed is weighed, and found to be 29·2 grammes. What is the specific heat of the iron?

Let c=specific heat of iron.

The latent heat of water is 79 (*vide* below)—that is, a gramme of ice at 0° C. in melting to water at 0° C. absorbs 79 units; and, therefore, 29·2 grammes will absorb $29{\cdot}2 \times 79$ units.

The heat given out by 200 grammes of iron in cooling from 100° C. to 0° C. will be $200 \times 100 \times c$ units.

And since—

$$(\text{heat lost}) = (\text{heat gained}),$$

$$\therefore 200 \times 100 \times c = 29{\cdot}2 \times 79$$

$$20{,}000c = 2{,}307$$

$$c = \frac{2{,}307}{20{,}000} = 0{\cdot}1153.$$

LATENT HEAT.

The latent heat of a substance is the heat absorbed (or given out) by a unit of a body in changing its state or condition (*e.g.*, from ice at 0° C. to water at 0° C.).

Example:

10 grammes of ice are mixed with 50 grammes of water at 29° C.; the resulting temperature is 11° C. Find the latent heat of water.

Let L=latent heat of water.

Then, heat required to melt 10 grammes of ice into

water at 0° C. $=10 \times L$, and heat required to raise melted ice from 0° C. to 11° C. $=10 \times 11=110$ units;
 $\therefore$ total heat absorbed by ice $=(10L+110)$ units.

Heat lost by 50 grammes of water in falling from 29° C. to 11° C. (*i.e.*, through 18 degrees) $=50 \times 18=900$ units.

$$\text{(heat gained)} = \text{(heat lost)}.$$
$$\therefore 10L+110=900$$
$$10L=900-110=790$$
$$L=79$$

That is, the change of 1 gramme of ice into 1 gramme of water at the same temperature, requires as much heat as will raise 1 gramme of water through 79° C.

SPECIFIC GRAVITY.

The specific gravity of a body is the number which expresses the relationship between the weight of a given volume of this body and the weight of the same volume of distilled water at 4° C. That is:

$$\text{S.G.} = \frac{\text{weight of a fixed vol. (in air)}}{\text{weight of equal vol. of water}}.$$

A. Solids.

The 'principle of Archimedes' affirms that a body immersed in a liquid loses a part of its weight equal to the weight of the displaced liquid. But the volume of this displaced liquid is, obviously, the volume of the body which displaces it; therefore it follows that a body immersed in a liquid loses weight equal to the weight of its own volume of the liquid. If this loss of weight be ascertained, then the weight of an equal volume of the liquid is also known. Therefore, the specific gravity of a body may be represented thus:

$$\text{S.G.} = \frac{\text{weight of the body in air}}{\text{loss of weight when weighed in water}}.$$

[For strict accuracy, the weighing should be performed *in vacuo*, to obviate the necessity of making any correction for the weights of the unequal volumes of air displaced by the substance in one scale-pan, and the weights in the other.]

B. Liquids.

A body is weighed successively in air, water, and the liquid whose specific gravity is to be ascertained. The loss of weight when weighed in water = weight of an equal volume of water. The loss of weight when weighed in the liquid = weight of an equal volume of the liquid; and since by the definition—

$$\text{S.G. of liquid} = \frac{\text{weight of a vol. of the liquid}}{\text{weight of an equal vol. of water}};$$

$$\therefore \text{S.G.} = \frac{\text{loss of weight when weighed in the liquid}}{\text{loss of weight when weighed in water}}.$$

Example:

Six cubic inches of zinc weigh 24·8 ounces. What is the specific gravity of zinc?

1,728 cub. in. (or 1 cub. ft.) of water weigh 1,000 oz.;

$$\therefore \text{6 cub. in. weigh } \frac{1{,}000 \times 6}{1{,}728} = 3{\cdot}472 \text{ oz.}$$

and 6 cub. in. of zinc weigh 24·8 oz.;

$$\therefore \text{S.G. of zinc} = \frac{24{\cdot}8}{3{\cdot}472} = 7{\cdot}15.$$

Example:

What is the specific gravity of a body of which n cubic feet weigh x lbs.?

1 cub. ft. of water weighs 1,000 oz. $=\frac{1,000}{16}$ lbs.;

$\therefore$ n cub. ft. of water weigh $\frac{1,000\,n}{16}$ lbs.,

and n cub. ft. of the body weigh x lbs.,

$$\therefore \text{S.G.} = \frac{x}{\frac{1,000\,n}{16}} = \frac{x \times 16}{1,000\,n} = \frac{0{\cdot}016\,x}{n}.$$

Example:

An ivory ball weighs 8·53 grammes in air, 4·07 grammes in water, and 3·93 grammes in milk. Find the specific gravity of (i.) the ivory, (ii.) the milk.

(i.) In water the ball loses $8{\cdot}53 - 4{\cdot}07 = 4{\cdot}46$ grammes;

$$\therefore \text{S.G. of ivory} = \frac{8{\cdot}53}{4{\cdot}46} = 1{\cdot}91.$$

(ii.) In water the ball loses 4·46 grammes;

$\therefore$ 4·46 grammes = weight of an equal vol. of water.

In milk the ball loses $8{\cdot}53 - 3{\cdot}93 = 4{\cdot}6$ grammes;

$\therefore$ 4·6 grammes = weight of an equal vol. of milk.

$$\therefore \text{S.G. of milk} = \frac{4{\cdot}6}{4{\cdot}46} = 1{\cdot}032$$

(or, in the more usual notation, 1032).

Example:

A piece of copper weighs 10 pounds in air and $8\frac{7}{8}$ pounds in water. Find its specific gravity and its volume in cubic inches (a cubic foot of water weighing 1,000 ounces).

$10 - 8\frac{7}{8} = 1\frac{1}{8}$ lbs. = 18 oz. = loss of weight in water.

1,000 oz. = 1 cub. ft. = 1,728 cub. in.;

$$\therefore 18 \text{ oz.} = \frac{1,728 \times 18}{1,000} = 31 \cdot 1 \text{ cub. in.}$$

Since loss of weight = 18 oz.,

∴ weight of displaced water = 18 oz.;

∴ vol. of displaced water = 31·1 cub. in.;

∴ vol. of piece of copper = 31·1 cub. in.;

$$\text{and S.G. of copper} = \frac{\text{weight in air}}{\text{loss of weight when weighed in water}}$$

$$= \frac{10}{1\frac{1}{8}} = 8 \cdot 9.$$

One c.c. of water weighs 1 gramme; the specific gravity of copper is 8·9—that is, a piece of copper is 8·9 times heavier than the same volume of water; therefore 1 c.c. of copper weighs 8·9 grammes. Similarly, the specific gravity of lead is 11·347; therefore 1 c.c. of lead weighs 11·347 grammes. For convenience of calculation, therefore, it may be remembered that *the weight of 1 c.c. of any substance is its specific gravity expressed in grammes.*

Example:

A piece of wood, the specific gravity of which is 0·6, weighs *in vacuo* 10 grammes. To it is attached a sinker weighing *in vacuo* 13·3 grammes; the specific gravity of the sinker is 11·347. What volume of water, expressed in cubic centimetres, will the two bodies displace when immersed in that fluid ?*

Firstly, with regard to the wood:

$$0{\cdot}6 \text{ grm. wood} = 1 \text{ c.c.}$$

$$\therefore 10 \text{ grms. wood} = \frac{10}{0{\cdot}6} = 16{\cdot}7 \text{ c.c.}$$

Since the volume of the wood measures 16·7 c.c., it will, when immersed, displace 16·7 c.c. of water.

Similarly, for the sinker:

$$11{\cdot}347 \text{ grms. sinker} = 1 \text{ c.c.}$$

$$\therefore 13{\cdot}3 \text{ grms. sinker} = \frac{13{\cdot}3}{11{\cdot}347} = 1{\cdot}17 \text{ c.c.}$$

That is, the sinker, when immersed, will displace 1·17 c.c. of water.

Total water displaced by wood and sinker will therefore be:

$$16{\cdot}7 + 1{\cdot}17 = 17{\cdot}87 \text{ c.c.}$$

As a corollary to the above problem, the size of the piece of wood required to support the sinker in the water may be calculated.

When a body is immersed in a fluid, the upward pressure on it is equal to the weight of fluid displaced by the body. The downward pressure, due to gravity, is equal to the weight of the body. Where there is equilibrium, these opposing forces must be equal to each other.

* D.P.H. Exam (Roy. Coll. Phys. Surg.).

Weight of sinker = 13·3 grammes,

∴ *downward* pressure on sinker = 13·3 grammes.

Volume of sinker = 1·17 c.c.,

∴ vol. of displaced water = 1·17 c.c.,

∴ weight of displaced water = 1·17 grammes,

∴ *upward* pressure on sinker = 1·17 grammes.

Let x = number of c.c. of wood required.

Then x c.c. wood weigh 0·6 x grammes;

that is, weight of wood = 0·6 x grammes;

∴ *downward* pressure on wood = 0·6 x grammes.

Vol. of wood = x c.c.;

∴ vol. of displaced water = x c.c.;

∴ weight of displaced water = x grammes;

∴ *upward* pressure on wood = x grammes.

And since

(upward pressure) = (downward pressure);

$$\therefore 1{\cdot}17 + x = 13{\cdot}3 + 0{\cdot}6\,x$$

$$x - 0{\cdot}6\,x = 13{\cdot}3 - 1{\cdot}17$$

$$0{\cdot}4\,x = 12{\cdot}13$$

$$x = 30{\cdot}3 \text{ c.c.}$$

Therefore, wood required to float the sinker = 30·3 c.c., or 30·3 × 0·6 = 18·18 grammes.

HYDRAULIC PRESS.

Pascal's law states that 'pressure exerted anywhere upon a mass of liquid is transmitted undiminished in all directions, and acts with the same force on all equal surfaces, and in a direction at right angles to those surfaces.'

Example:

The smailer piston of a Bramah's Hydraulic Press is ½ inch in diameter, the larger piston is 10 inches in diameter; a weight of 12 lbs. is placed on the small. What will be the weight placed on the larger?*

* D.P.H. Exam. (Roy. Coll. Phys. Surg.).

$$\text{Area of circle} = \frac{\pi}{4} \times d^2 \text{ (p. 178).}$$

$$\text{Diameter of small piston} = \tfrac{1}{2} \text{ inch;}$$

$$\therefore \text{ area of small piston} = \frac{\pi}{4} \times (\tfrac{1}{2})^2.$$

$$\text{Diameter of large piston} = 10 \text{ inches;}$$

$$\therefore \text{ area of large piston} = \frac{\pi}{4} \times (10)^2;$$

$$\therefore \left\{\begin{matrix}\text{area of}\\ \text{small piston}\end{matrix}\right\} : \left\{\begin{matrix}\text{area of}\\ \text{large piston}\end{matrix}\right\} :: \frac{\pi}{4} \times (\tfrac{1}{2})^2 : \frac{\pi}{4} \times (10)^2$$

$$:: (\tfrac{1}{2})^2 : (10)^2$$

$$:: \tfrac{1}{4} : 100$$

$$:: 1 : 400;$$

That is, the area of the large piston is 400 times that of the small. The surface of the large piston may thus be considered to be made up of 400 units of surface, each unit having the same area as the small piston. And since a weight of 12 lbs. is placed upon the small piston, this weight will be transmitted undiminished (according to Pascal's law) to the water, and to each of the 400 units on the surface of the large one. The pressure on the larger piston, therefore, will be—

$$12 \times 400 = 4{,}800 \text{ lbs.} = 2{\cdot}14 \text{ tons.}$$

CHAPTER IV

VENTILATION

THE greater the velocity of the air, the greater will be the outflow, or amount delivered, in a given time; that is, velocity varies directly as outflow.

Also, velocity varies inversely as the sectional area. This is not always obvious at first, but on a little consideration it will be seen to be true.

Take, by way of example, a narrow river suddenly widening its bed for a distance, and then narrowing again. Exactly the same quantity of water which enters the widened portion must leave it (for if more entered than left, the water would be dammed back; if more left than entered, the widened portion would run dry). Therefore, the same amount of water must pass the widened portion as passes an equal area of the narrow portion in the same time; but as the bed is widened, the velocity must be diminished in order to fulfil this condition. So the greater the sectional area, the less need the velocity be, in order to deliver a given quantity in a given time; on the other hand, the less the sectional area, the greater must be the velocity, in order to deliver the required amount in the time. Therefore, since the velocity varies directly as the outflow, and inversely as the sectional area, it may be represented as follows:

$$\text{Velocity} = \frac{\text{outflow}}{\text{sectional area}};$$

or, more briefly,

$$V = \frac{O}{S}.$$

In the Delivery of Air through an Inlet, having given Two of the Three Following Data (viz., Velocity, Delivery, and Sectional Area), to find the Third.

Examples:

(i.) Find the velocity required to deliver 3,000 cubic feet of air per hour through an inlet, whose sectional area is 24 square inches.

O = 3,000 cubic feet per hour, and S = 24 square inches, or $\frac{24}{144} = \frac{1}{6}$ square foot.

$$\therefore V = \frac{O}{S} = \frac{3{,}000 \text{ cubic feet}}{\frac{1}{6} \text{ square foot}} = 18{,}000 \text{ feet per hour,}$$

$$\text{or } \frac{18{,}000}{60 \times 60} = 5 \text{ feet per second}$$

(or 3·4 miles per hour).

(ii.) Find sectional area of inlet required to deliver 3,000 cubic feet of air per hour, with a velocity of 5 feet per second.

O = 3,000 cubic feet per hour.

V = 5 feet per second, or 18,000 feet per hour.

$$V = \frac{O}{S}; \therefore S = \frac{O}{V} = \frac{3{,}000 \text{ cubic feet}}{18{,}000 \text{ feet}} = \frac{1}{6} \text{ square foot,}$$

$$= \frac{144}{6} = 24 \text{ square inches.}$$

(iii.) An inlet, having a sectional area of 24 square inches, delivers air at the rate of 5 feet per second; find total delivery per hour.

$$V = 5 \text{ feet per second.}$$

$$S = 24 \text{ square inches} = \frac{24}{144} = \frac{1}{6} \text{ square foot.}$$

$$V = \frac{O}{S}; \therefore O = V \times S = 5 \text{ feet} \times \frac{1}{6} \text{ square foot;}$$

$$= \frac{5}{6} \text{ cubic foot per second;}$$

$$= \frac{5 \times 60 \times 60}{6} = \left\{ \begin{array}{l} 3{,}000 \text{ cubic feet} \\ \text{per hour.} \end{array} \right.$$

For the sake of simplicity, the same figures have been taken in each of the above three examples, from which it is seen that an inlet whose area is 24 square inches will deliver 3,000 cubic feet of air per hour, with a velocity of 5 feet per second.

There are one or two points to notice in the solution of the above examples. In (i.) and (iii.) one of the data given is in *inches*, the other in *feet*. These must, of course, be both brought either to inches or feet before proceeding further. In (ii.) O = feet per *hour*, V = feet per *second*. Here, one of them must be brought to the same terms as the other.

Also it may be pointed out that:

$$\frac{\text{cubic feet}}{\text{square feet}} = \text{feet}; \quad \frac{\text{cubic feet}}{\text{feet}} = \text{square feet};$$

$$\frac{\text{square feet}}{\text{feet}} = \text{feet}; \quad \text{feet} \times \text{square feet} = \text{cubic feet.}$$

It will be noticed that no allowance has been made for friction (for further reference to this subject, *vide* p. 61). It is usual, for an ordinary inlet, to deduct one-fourth for friction.

Thus, in example (i.) 5 feet per second becomes $5 - \frac{5}{4} = 3{\cdot}75$ feet per second; and this lessening of the velocity will lessen the total delivery by one-fourth, which would then be only $3{,}000 - \frac{3{,}000}{4} = 2{,}250$ cubic feet per hour.

With the lessened velocity, the same-sized inlet (viz., 24 square inches) would suffice to give the proportionately diminished delivery; but, in order to produce the 3,000 cubic feet per hour, allowing the velocity to be diminished by friction, the inlet would have to be enlarged in the same proportion as the increased delivery required; thus,

$$2250 : 3000 :: 24 \text{ sq. in.} : x \text{ sq. in.};$$

from which we find that $x = 32$ square inches.

The above methods do not take into consideration the difference (if any) in the temperature of the inside and outside air, nor yet the difference in level between inlet and outlet. (In this case the velocity must be calculated by Montgolfier's formula, p. 56.)

The following is a simple method of calculating the total delivery and velocity of the air, and the sectional area of the inlet.

If the sectional area be 1 square foot and the velocity 1 foot per second, the delivery must obviously be 1 cubic foot per second.

Sectional Area of Opening.	*Velocity.*	*Delivery.*
1 sq. ft., or 144 sq. in.	1 ft. per sec. ...	1 cub. ft. per sec., or 3,600 cub. ft. per hour.
ditto ...	5 ft. per sec. ...	$5 \times 3{,}600 = 18{,}000$ cub. ft. per hour.
$\frac{144}{6} = 24$ sq. in.	ditto ...	$\frac{18{,}000}{6} =$ 3,000 cub. ft. per hr.

To Calculate the Amount of CO_2 expired by an Adult per Hour.

Inspired air contains 4 parts per 10,000, or 0·04 per cent. of CO_2. Expired air contains 4·04 per cent.;

$\therefore$ 4·04 − 0·04 = 4 per cent. of CO_2 is given off in each breath.

In the case of an adult male, at each breath 30·5 cubic inches of air pass in and out of the lungs, containing, when expired, an additional 4 per cent.,

$$\text{or, } \frac{4 \times 30 \cdot 5}{100} = 1 \cdot 22 \text{ cubic inches } CO_2.$$

17 respirations per minute, or 1,020 respirations per hour, would produce $1 \cdot 22 \times 1020 = 1244 \cdot 4$ cubic inches per hour,

$$\text{or, } \frac{1244 \cdot 4}{1728} = 0 \cdot 72 \text{ cubic foot per hour.}$$

Women and children exhale less than this, and 0·6 cubic foot per hour is about the average per head for a mixed assembly of people.

To Calculate the Necessary Air-supply and the Impurity Present.

Let D = delivery of air (in cubic feet), or amount of air available.

E = total amount of CO_2 exhaled.

r = *added* respiratory impurity in 1 cubic foot of air.

Then,

$$\left.\begin{matrix}\text{added im-}\\ \text{purity in}\\ \text{1 cub. ft.}\end{matrix}\right\} : \left\{\begin{matrix}\text{1 cub.}\\ \text{ft. of}\\ \text{air.}\end{matrix}\right\} :: \left\{\begin{matrix}\text{total im-}\\ \text{purity}\\ \text{added}\end{matrix}\right\} : \left\{\begin{matrix}\text{total de-}\\ \text{livery of}\\ \text{air.}\end{matrix}\right.$$

$$\therefore r : 1 :: E : D;$$

$$\therefore D = \frac{E}{r}.$$

The following data should be borne in mind:

On an average, each person exhales 0·6 cubic foot of CO_2 per hour, and this figure is usually taken in all calculations.

The total CO_2 in a room should not exceed 0·6 part per 1,000; and since atmospheric air contains 0·4 part per 1,000, it follows that the amount of *added* respiratory impurity should not exceed 0·6 – 0·4 = 0·2 part per 1,000.

How much fresh air should each person be allowed per hour, in order that the above conditions may be fulfilled?

$$E = 0{\cdot}6 \text{ cubic foot per hour};$$

$$r = 0{\cdot}2 \text{ per } 1{,}000, \text{ or } 0{\cdot}0002 \text{ per cubic foot};$$

$$\therefore D = \frac{E}{r} = \frac{0{\cdot}6}{0{\cdot}0002} = 3{,}000.$$

That is, each person should be allowed 3,000 cubic feet of fresh air per hour.

Example:

In a room 20 feet long × 15 feet wide × 8 feet high, occupied by three persons, state how many cubic feet of fresh air per hour would be required; and what should be the aggregate dimensions of inlets, so that the rate of in-flow of air should not exceed four linear feet per second.*

* D.P.H. Exam., Cambridge.

Room $= 20 \times 15 \times 8 = 2{,}400$ cub. feet.

Three persons exhale $0{\cdot}6 \times 3 = 1{\cdot}8$ cub. feet CO_2 per hour.

Permissible added impurity $= 0{\cdot}0002$ cub. feet.

$$\therefore \; D = \frac{E}{r} = \frac{1{\cdot}8}{0{\cdot}0002} = \frac{18{,}000}{2} = 9{,}000 \text{ cub. feet per hour.}$$

(For the first hour $9{,}000 - 2{,}400 = 6{,}600$ cub. feet.)

4 feet per sec. $= 240$ feet per min. $= 14{,}400$ feet per hour.

$$S = \frac{O}{V} = \frac{9{,}000}{14{,}400} = \frac{5}{8} \text{ sq. feet} = \frac{5 \times 144}{8} = 90 \text{ sq. inches.}$$

So, 9,000 cub. feet of air will be required per hour, and 90 sq. inches of inlet space.

Example :

The air of a room containing 20,000 cubic feet, in which 10 persons have been working for 5 hours, is found to contain 10 parts of carbonic acid in 10,000 parts. How much fresh air is entering per head per hour?*

Air of room yields 10 parts CO_2 per 10,000; CO_2 naturally present in air $= 4$ parts per 10,000; therefore the added respiratory impurity $= 10 - 4 = 6$ parts per 10,000, or $0{\cdot}0006$ cubic foot of CO_2 per cubic foot of air; that is,

$$r = 0{\cdot}0006.$$

Ten persons exhale $10 \times 0{\cdot}6 = 6$ cubic feet per hour, or 30 cubic feet in five hours;

$$\therefore \; E = 30.$$

$$D = \frac{E}{r} = \frac{30}{0{\cdot}0006} = 50{,}000 \text{ for the five hours,}$$

or 10,000 per hour for the 10 persons—
that is, 1,000 cubic feet per head per hour.

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* D.P.H. Exam. (Roy. Coll. Phys. Surg.).

Since total delivery per hour = 10,000 cubic feet, and the room contains 20,000 cubic feet of space, there is sufficient air in the room for the first two hours, but afterwards 10,000 cubic feet per hour (or 1,000 cubic feet per head) must be supplied in order to keep the CO_2 down to the stated limit of 10 parts per 10,000.

Example:

(i.) Six persons occupy a room of 5,000 cubic feet space continually for six hours; calculate the percentage of CO_2 present in the air at the end of the time, supposing 10,000 cubic feet of fresh air have been supplied per hour.

(ii.) In what time would the permissible limit of impurity be reached if there were no ventilation?

(i.) Six persons exhale $0{\cdot}6 \times 6 = 3{\cdot}6$ cubic feet CO_2 per hour, or $3{\cdot}6 \times 6 = 21{\cdot}6$ cubic feet CO_2 in 6 hours;

$$\therefore E = 21{\cdot}6.$$

10,000 cubic feet of air supplied per hour = 60,000 cubic feet in 6 hours, which with the 5,000 cubic feet originally present = 65,000 cubic feet;

$$\therefore D = 65{,}000.$$

$$D = \frac{E}{r};\ \therefore r = \frac{E}{D} = \frac{21{\cdot}6}{65{,}000} = 0{\cdot}00033 \text{ per cubic foot,}$$

or $0{\cdot}033$ per cent. of added impurity.

Add to this the CO_2 originally present in air—viz., $0{\cdot}04$ per cent.—

$\therefore$ Total impurity $= 0{\cdot}033 + 0{\cdot}04 = 0{\cdot}073$ per cent.

[Problems of this nature may also be solved without the use of the formula, as follows:

CO_2 originally present = 0·4 per 1,000, or 2 cubic feet per 5,000.

CO_2 exhaled, as has been already seen = 21·6 cubic feet.

The fresh air added (10,000 per hour for 6 hours) = 60,000 cubic feet, which naturally contain 4 cubic feet CO_2 per 10,000, or 24 cubic feet per 60,000.

Therefore total CO_2 at end of 6 hours =

Originally present	2·0
Added by respiration	21·6
Added in fresh air	24·0
Total	47·6

The amount of air available has been seen to be 65,000 cubic feet. Therefore, total amount of CO_2 present at end of 6 hours = 47·6 cubic feet in 65,000 cubic feet of air,

$$\text{or, } \frac{47{\cdot}6 \times 100}{65{,}000} = 0{\cdot}073 \text{ per cent.]}$$

(ii.) In what time would the permissible limit be reached, if there were no ventilation?

The permissible total impurity is 0·6 per 1,000 (p. 49),

or $0{\cdot}6 \times 5 = 3$ cubic feet in the 5,000.

The room already contains 0·4 per 1,000,

or $0{\cdot}4 \times 5 = 2$ cubic feet in the 5,000.

Therefore, the permissible *added* impurity $= 3 - 2 = 1$ cubic foot.

How long will it take the six persons to add this 1 cubic foot?

In 1 hour 6 persons exhale $0{\cdot}6 \times 6 = 3{\cdot}6$ cubic feet CO_2;

$\therefore$ in $\frac{1}{3{\cdot}6}$ hour they will add 1 cubic foot CO_2,

$$\text{and } \frac{1}{3{\cdot}6} \text{ hour} = \frac{60}{3{\cdot}6} = 16{\cdot}6 \text{ minutes.}$$

[This result may also be obtained as follows :

Let x = time (in hours) ;

CO_2 in room = 2 cubic feet ;

Six men in x hours add $3{\cdot}6x$ cubic feet ;

$\therefore$ Total CO_2 in x hours = $(3{\cdot}6x + 2)$ cubic fect.

Available air = 5,000 cubic feet,

And permissible limit of impurity = 6 per 10,000.

Since this limit must not be surpassed, the total CO_2 and the available air must be to each other in the proportion of 6 to 10,000, or

$$\text{Total } CO_2 : \text{available air} :: 6 : 10{,}000;$$

$$\therefore 3{\cdot}6x + 2 : 5{,}000 :: 6 : 10{,}000;$$

$$3{\cdot}6x + 2 = \frac{5{,}000 \times 6}{10{,}000} = 3;$$

$$\therefore x = \frac{3-2}{3{\cdot}6} = \frac{1}{3{\cdot}6} \text{ hours} = 16{\cdot}6 \text{ minutes.]}$$

Example :

A room 5,000 cubic feet capacity is occupied by four people for 12 hours ; the carbon dioxide was found to be 1·2 parts per 1,000. How much fresh air per hour has been supplied to each person ?*

[Ans. : 750 cubic feet per head per hour.]

Example :

A room is ventilated by four equal inlets having an aggregate sectional area of 48 square inches, is occupied by two adult persons, and is lighted by one No. 5 bat's-wing burner consuming 6·5 cubic feet of oxygen, and producing 2·8 cubic feet of CO_2 per hour. Find the velocity of the air-current per minute through each inlet needful to keep down the impurity to the permissible limit.†

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* D.P.H. Exam. (Roy. Coll. Phys. Surg.).

† D.P.H. Exam., Cambridge.

The necessary supply of air for the gas-burner should be regulated by the impurity it produces, rather than by the amount of oxygen it consumes.

Two persons exhale $2 \times 0 \cdot 6 = 1 \cdot 2$ cub. feet CO_2 per hour.
One gas-burner produces $2 \cdot 8$,, ,, ,,

Total $CO_2 = 4 \cdot 0$ cub. feet per hour.

The permissible limit of added impurity $= 0 \cdot 2$ parts of CO_2 per 1,000 or $0 \cdot 0002$ per cubic foot.

$$D = \frac{E}{r} = \frac{4}{0 \cdot 0002} = \frac{40{,}000}{2} = 20{,}000.$$

That is, 20,000 cub. feet of fresh air are required per hour.

Sectional area of the 4 tubes $= 48$ sq. in. $= \frac{48}{144} = \frac{1}{3}$ sq. feet.

$$V = \frac{O}{S} \text{ (p. 45)} = \frac{20{,}000}{\frac{1}{3}} = 60{,}000 \text{ linear feet per hour.}$$
$$= 1{,}000 \quad \text{,,} \quad \text{,,} \quad \text{per minute.}$$

[NOTE.—Although the velocity through *each* inlet is required, the estimated velocity—viz., 1,000 linear feet per minute—must not be divided by 4. The fact that there are four inlets simply means that (*if the question of friction be neglected*) one-fourth of the total air-supply will be delivered through each of the four inlets with the same velocity as the whole air-supply would be through one inlet of four times the size. In one case $V = \frac{O}{S}$, and in the other $V = \frac{\frac{1}{4} \cdot O}{\frac{1}{4} \cdot S} = \frac{O}{S}$.]

IMPURITY PRODUCED BY ARTIFICIAL ILLUMINATION.

Combustion of Oil.

It is estimated that a lamp with one moderately good wick burns 154 grains of oil per hour. Oil (paraffin) contains about 85 per cent. of carbon. That is—

100 grs. oil contain 85 grs. C., and 154 grs. oil contain

$$\frac{85 \times 154}{100} = 130 \cdot 9 \text{ grs. C.}$$

The equation $C_2 + 2O_2 = 2CO_2$ shows that 24 grs. C. produce 88 grs. CO_2;

$$\therefore 130 \cdot 9 \text{ grs. C. produce } \frac{88 \times 130 \cdot 9}{24} = 480 \text{ grs. } CO_2;$$

$$\therefore 154 \text{ grs. oil produce } 480 \text{ grs. } CO_2.$$

Now, 1 cub. foot of dry air at 60° F. (the average temperature of a room) and 760 mm. weighs

$$\frac{566 \cdot 85 \times 491}{491 + (60 - 32)} = 536 \cdot 27 \text{ grs. (p. 11).}$$

$\therefore$ 1 cub. foot CO_2 (at same temperature and pressure)

$$\text{weighs } \frac{536 \cdot 27 \times 22}{14 \cdot 47} = 815 \cdot 3 \text{ grs. (p. 5).}$$

So, if 815·3 grs. $CO_2 = 1$ cub. foot, then

$$480 \text{ grs. } CO_2 = \frac{480}{815 \cdot 3} = 0 \cdot 59 \text{ cub. feet.}$$

Therefore 154 grs. of oil produce 0·59 cub. feet of CO_2.

That is, the combustion of the oil in the lamp produces 0·59 cubic foot, or rather more than $\frac{1}{2}$ cubic foot of CO_2 per hour. And since a human being exhales 0·6 cubic foot per hour, it will be seen that an ordinary lamp produces, each hour, about the same vitiation of the air, in respect to the CO_2, as each occupant of the room.

Example:

A room is fitted with four Tobin's tubes, each having a sectional area of 50 square inches, and in the room are two persons and a paraffin lamp, wh ch burns 1 ounce of oil per hour. Find the velocity of the air-current necessary to keep the impurity down to 0·6 part per 1,000.

154 grains oil produce 0·59 cubic foot CO_2 (p. 55):

$\therefore$ 1 ounce oil produces $\frac{0·59 \times 437·5}{154} = 1·6$ cubic feet CO_2 per hour.

Two persons exhale $2 \times 0·6 = 1·2$ cubic feet CO_2 per hour;

$\therefore$ total CO_2 produced $= 1·6 + 1·2 = 2·8$ cubic feet.

Total permissible amount of $CO_2 = 6$ per 10,000.

Amount present in air $= 4$ per 10,000.

$\therefore$ permissible respiratory impurity $= 6 - 4 = 2$ per 10,000; or, 0·0002 per cubic foot.

$$D = \frac{E}{r} = \frac{2·8}{0·0002} = 14,000 \text{ cubic feet per hour;}$$

$\therefore$ each 'Tobin' must deliver $\frac{14,000}{4} = 3,500$ cubic feet per hour.

$$\text{Now, } V = \frac{O}{S} \text{ (p. 45)} = \frac{3,500 \text{ cubic feet}}{50 \text{ square inches}} = \frac{3,500 \text{ cubic feet}}{\frac{50}{144} \text{ square feet}}$$

$= 10,080$ feet per hour, or 2·8 feet per second.

[No deduction has been here made for friction.]

MONTGOLFIER'S FORMULA.

When bodies fall to the earth, we find that different bodies fall through equal spaces from rest, in a given time; and the space fallen through varies as the square of the time.

Any deviation from this law is due to the resistance of the air.

If V = velocity in feet per second,
g = acceleration due to gravity,
h = height fallen (in feet),
then $V^2 = 2gh$.

The value of g increases from the equator to the poles; in the latitude of London, $g = 32{\cdot}19$ feet per second. For practical purposes, however, we may consider the value of g to be *constant*.

Suppose $h = 5$ miles $= 26{,}400$ feet,
Then $V^2 = 2gh = 2 \times 32{\cdot}19 \times 26{,}400 = 1{,}699{,}632$, and
$V = \sqrt{1{,}699{,}632} = 1{,}303$ feet per second.

If the atmosphere be assumed to be of uniform density throughout, it would extend upwards to a height of about five miles; and the velocity with which air enters a vacuum is the same as a body would acquire in falling through a corresponding height; therefore the velocity of air entering a vacuum $= 1{,}303$ feet per second. If the air be not entering a vacuum, but a space containing air at a different pressure, its velocity will depend on the difference between their respective pressures. Usually these pressures cannot be directly observed, but must be inferred from the difference between their temperatures. The difference in pressure can be estimated by ascertaining the difference in level between inlet and outlet, and multiplying this by the expansion of air due to the difference in temperature; and this difference of pressure may be taken to represent the height fallen.

The equation $V^2 = 2gh$ may be expressed as follows:

$V^2 = 2gh = 2 \times 32{\cdot}19 \times h = 64{\cdot}38 \times h$;

$\therefore V = \sqrt{64{\cdot}38} \times \sqrt{h} = 8{\cdot}02 \times \sqrt{h}$

(approximately $8\sqrt{h}$).

That is, the velocity in feet per second of falling bodies $= 8 \times \sqrt{\text{height fallen}}$.

Example :

Assuming that two adults sleep in the room (whose dimensions are given in the example on p. 181), how much inlet and outlet area per head must be given to supply each person with 1,500 cubic feet of air hourly, supposing that the outside air temperature is 40° F., and the internal air temperature is 60° F., and the height of the heated column of air is 20 feet?*

Since difference in level between inlet and outlet $= 20$ feet, and expansion of air due to increase of temperature of $(60-40=)$ 20° F. $= \frac{20}{491}$,

$$\therefore \text{height fallen} = 20 \times \frac{20}{491} = \frac{400}{491} \text{ feet,}$$

and $V = 8\sqrt{h} = 8\sqrt{\frac{400}{491}} = 8 \times \frac{20}{\sqrt{491}} = 7{\cdot}22$ feet per second

$= 25{,}992$ feet per hour.

Deduct $\frac{1}{4}$ for friction, then

$$V = 25{,}992 - \frac{25{,}992}{4} = 19{,}494 \text{ feet per hour.}$$

Since delivery required $= 1{,}500$ cubic feet per head per hour, and velocity $= 19{,}494$ linear feet per hour, the sectional area of the inlet will be (p. 45) :

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* D.P.H. Exam., Cambridge.

$$S = \frac{O}{V} = \frac{1,500}{19,494} \text{ sq. feet,}$$

$$= \frac{1,500 \times 144}{19,494} = 11 \text{ sq. inches per head,}$$

and there ought also to be an outlet of equal size.

The cubic capacity of the room—viz., 2,160 cubic feet—need not be considered, since the necessary air-supply is given in the question, and therefore has not to be estimated either from the size of the room, or from the amount of impurity present.

Example:

Explain the formula—

$$\text{Velocity} = \sqrt{2g \times \text{height } (T - t) \times 0{\cdot}002}$$

with reference to an air extraction shaft.

The temperature of the air in a perpendicular ventilation shaft of $2 \times 1\frac{1}{2}$ feet section being 80° F., its length 30 feet, and the temperature of the external air 50° F., what quantity of air per hour would the shaft be capable of delivering?*

This formula is in reality $V = 8\sqrt{h}$, which has been already explained, for $\sqrt{2g}$ has been shown to be 8; 'height' = difference in level between inlet and outlet (30 feet); $T - t$ = difference between internal and external temperature (30° F.); and $0{\cdot}002 = \frac{1}{491}$ approximately.

$$\therefore \text{Velocity} = 8\sqrt{30 \times 30 \times 0{\cdot}002},$$

$$= 8\sqrt{1{\cdot}8},$$

$$= 10{\cdot}73 \text{ feet per second,}$$

$$= 38,628 \text{ feet per hour.}$$

* D.P.H. Exam., Cambridge.

Deduct $\frac{1}{4}$ for friction—

$$38{,}628 - \frac{38{,}628}{4} = 28{,}971 \text{ feet per hour.}$$

Sectional area of shaft $= 2 \times 1\frac{1}{2} = 3$ sq. feet ;

$$\therefore \ O = V \times S = 28{,}971 \times 3 = 86{,}913 \text{ cub. feet per hour.}$$

Example :

(i.) A room 30 feet long, 20 feet wide, and 12 feet high, containing 3 gas-burners, is occupied by 5 persons. If there be no ventilation, in what time will the air have reached its permissible limit of impurity? (ii.) What delivery of air is required to prevent the permissible limit being exceeded? (iii.) If the only outlet be a chimney 15 feet high, and the temperature of the room 10° F. higher than the outside air, find the sectional area of the chimney required.

(i.) Five persons exhale $5 \times 0{\cdot}6 = 3$ cubic feet CO_2 per hour. 1 cubic foot of gas produces about 0·6 cubic foot CO_2 ; a gas-burner consumes about 5 cubic feet of gas per hour ; therefore each burner produces $5 \times 0{\cdot}6 = 3$ cubic feet CO_2 per hour, and 3 burners produce 9 cubic feet CO_2 ;

$$\therefore \text{ total } CO_2 = 3 + 9 = 12 \text{ cubic feet per hour ;}$$

$$\therefore \ E = 12 \text{ (p. 48).}$$

The room $= 30 \times 20 \times 12 = 7{,}200$ cubic feet ;

$$\therefore \ D = 7{,}200.$$

$$r = \frac{E}{D} = \frac{12}{7{,}200} = 0{\cdot}00167.$$

If at the end of hour $r=0{\cdot}00167$, how long will it take r to reach the permissible limit of 0·0002?

Let x=number of minutes; then

$$0{\cdot}00167 : 0{\cdot}0002 :: 60 \text{ minutes} : x \text{ minutes.}$$

$$\therefore x = 7{\cdot}2 \text{ minutes.}$$

(ii.) $D=\frac{E}{r}=\frac{12}{0{\cdot}0002}=60{,}000$ cubic feet per hour.

(iii.) Velocity in feet per second $=8\sqrt{\frac{15\times10}{491}}=4{\cdot}4216$ per second $=15{,}918$ feet per hour. After deducting $\frac{1}{4}$ for friction, this is reduced to 11,940 feet per hour.

$$S=\frac{O}{V}\text{ (p. 45)}=\frac{60{,}000}{11{,}940}=5 \text{ square feet.}$$

Therefore the chimney must have a sectional area of 5 square feet.

FRICTION IN VENTILATION.

Example:

Compare the ventilation in the two rooms, A and B.

The room A is ventilated by a straight shaft, circular in section, 30 feet long, with a sectional area of 1 square foot.

The room B is ventilated by four shafts, square in section, each 30 feet long, and each having a sectional area of $\frac{1}{4}$ square foot.

It will be noticed that the sum of the sectional areas of the four smaller shafts in B is equal to the sectional area of the larger shaft in A, and the lengths are the same—viz., 30 feet. Therefore, the cubic contents, or capacity of the four shafts in B, are together equal to the capacity of the larger shaft in A, and would be capable, therefore, of supplying the same quantity of air to B as the single shaft does to A, were it not for friction, which will now be considered.

In two tubes of similar shape and equal sectional area, the friction will vary directly as the length—*e.g.*, in two similar tubes, one 20 feet and the other 30 feet long, the friction in the longer tube will be increased by one-half, since its length is increased by one-half. If a shaft 40 feet long be increased $\frac{1}{4}$ of its length (*i.e.*, become 50 feet long), it will have an increased friction of $\frac{1}{4}$.

In two tubes of different shape, but of the same sectional area, the friction varies directly as the periphery.

For example, take a circular and a square tube, each with a sectional area of 1 square foot. The periphery of the circle, with a sectional area of 1 square foot or 144 square inches, may be found as follows (*vide* p. 178):

$$\pi r^2 = 144 \text{ sq. in.}; \therefore r^2 = \frac{144}{\pi} = \frac{144}{3{\cdot}1416} = 45{\cdot}8365;$$

$$\therefore r = \sqrt{45{\cdot}8365} = 6{\cdot}77;$$

$$\text{therefore, diameter} = 6{\cdot}77 \times 2 = 13{\cdot}5 \text{ inches.}$$

If diameter $= 13{\cdot}5$, then periphery $= 13{\cdot}5 \times 3{\cdot}1416 = 42$ inches, or $3\frac{1}{2}$ feet.

A square with sectional area of 1 square foot has sides 1 foot long, and, therefore, periphery $= 4$ feet.

Thus:

$$\left\{\begin{matrix}\text{Friction in} \\ \text{circular tube}\end{matrix}\right\} : \left\{\begin{matrix}\text{friction in} \\ \text{square tube}\end{matrix}\right\} :: 3\tfrac{1}{2} : 4.$$

To return to the comparison of the rooms:

In A, shaft $= 30$ feet, and periphery $= 3\frac{1}{2}$ feet; so total friction may be represented as:

$$30 \times 3\tfrac{1}{2} = 105 \text{ units.}$$

In B, the sectional area of the small square shafts being $\frac{1}{4}$ square foot, each side will be $\frac{1}{2}$ foot long, and the total periphery of the four sides $= 4 \times \frac{1}{2} = 2$ feet.

Each shaft $= 30$ feet in length, and periphery $= 2$ feet ;
∴ total friction in each shaft $= 30 \times 2 = 60$ units, and
total friction in the four shafts $= 60 \times 4 = 240$ units.

So the following result is obtained :

Friction in A : friction in B : : 105 : 240 ;
: : 7 : 16.

That is, the friction in B is more than twice that in A.

If, in each shaft in B, we place a right-angled bend, the friction will be doubled ; or,

A : B : : 7 : 16 × 2 ;
: : 7 : 32 ;

or friction in B is between four or five times that in A. Therefore, to get the same ventilation in B as in A, we must either increase the number of the smaller shafts, or the size of them, between four and five times if the bent tubes be used, or more than double them if the straight small shafts be used.

Example :

Suppose two ventilating shafts, the distance from the floor of the room to the point of exit being 50 feet in each case. Shaft A is straight, circular in section, with diameter of 8 inches. Shaft B is bent once at a right angle, cross-section is rectangular, measuring 10 inches by 5 inches, and the outside temperature is 50° F. ; the inside temperature in each case is 65° F. What are the respective volumes of air per hour which may be expected to pass out of the shafts ?*

D.P.H. Exam., Cambridge.

Since difference in level between inlet and outlet = 50 feet, and expansion of air for temperature

$$= \frac{65-50}{491} = \frac{15}{491},$$

$$\therefore h = 50 \times \frac{15}{491} = 1{\cdot}5275,$$

and $V = 8\sqrt{h} = 8\sqrt{1{\cdot}5275} = 8 \times 1{\cdot}2359 = 9{\cdot}8873$ feet per second, or 35,494 feet per hour.

Sect. area of $A = d^2 \times 0{\cdot}7854$ (p. 178) $= (8)^2 \times 0{\cdot}7854 =$ 50 sq. inches $= \frac{50}{144}$ sq. feet.

$$\therefore O = V \times S = 35{,}494 \times \frac{50}{144} = 12{,}324 \text{ cub. feet per hour.}$$

And sect. area of $B = 10 \times 5 = 50$ sq. inches.

The sectional areas of A and B are thus equal in size, their lengths are the same, and both are exposed to the same internal and external temperatures; but owing to their different shapes, and the bend in B, the delivery of air through them will be differently affected by friction. The periphery (or circumference) of the circular shaft is $8 \times 3{\cdot}1416 = 25$ inches. The periphery of the rectangular shaft having two sides of 10 inches, and two of 5 inches $= 30$ inches.

∴ Friction in circular shaft : friction in rectangular shaft :: 25 : 30.
:: 5 : 6.

The right-angled bend in B doubles the friction in it.

∴ Friction in A : friction in B :: 5 : 6 × 2.
:: 5 : 12.

Therefore, if A can deliver 12,324 cubic feet per hour, B will only deliver $\frac{5 \times 12{,}324}{12} = 5{,}135$ cubic feet per hour.

VENTILATION BY PROPULSION.

To Calculate Delivery of Air by a Revolving Fan.

The revolution of the fan sets the air in contact with it in motion. Each part of the fan is not, of course, revolving with the same velocity, which varies from a maximum at the extremities to *nil* at the centre. The velocity of the air is usually taken to be $\frac{3}{4}$ of that of the circumference of the fan, and is called the 'effective velocity.' So, if the speed of revolution of the extremity of the fan be known, the rate of movement of the air will be $\frac{3}{4}$ of this.

Example:

Suppose fan = 12 feet in diameter, then the circumference will be $12 \times 3{\cdot}1416 = 37{\cdot}70$ feet (*vide* p. 178). Therefore, in one complete revolution, the extremity of the fan travels 37·70 feet. If the fan be revolving 60 times per minute, the velocity of the extremity will be $37{\cdot}70 \times 60 = 2{,}262$ feet per minute, and the 'effective velocity' $= \frac{3 \times 2{,}262}{4} = 1{,}696$ feet per minute, or 101,760 feet per hour, which is the rate of movement of the air.

If the sectional area of the conduit, through which the air is delivered into the room, be known, then the discharge in cubic feet can be at once calculated by the formula $O = V \times S$ (p. 45).

By reversing this process, the size of the fan necessary to ventilate a room could be calculated, if the size of the conduit and the total delivery per hour of fresh air required were known. These two data would give the velocity per hour of the entering air, since

$$V = \frac{O}{S}.$$

Suppose this velocity to be 84,840 feet per hour; then the rotatory velocity of the fan must be such as to impart to the air a rate of movement of 84,840 feet per hour. That is, the 'effective velocity' = 84,840 feet per hour, or $\frac{3}{4}$ of the velocity at the extremity of the fan. Therefore,

$$\text{Velocity at extremity of fan} = \frac{4 \times 84{,}840}{3} = 113{,}120$$

feet per hour, or 1,885 feet per minute.

Now, if D = diameter of fan (in feet),
then $D \times 3{\cdot}1416$ = circumference of fan (in feet);
and if R = number of revolutions per minute,
then $D \times 3{\cdot}1416 \times R$ = distance travelled by circumference of fan per minute; but this = 1,885;

$$\therefore\ D \times 3{\cdot}1416 \times R = 1{,}885;$$

$$\therefore\ D \times R = \frac{1{,}885}{3{\cdot}1416} = 600 \text{ feet};$$

$$\therefore\ D = \frac{600}{R}, \text{ and } R = \frac{600}{D};$$

So, if D = 5, then R = 120;
if D = 3, then R = 200;
if D = 2, then R = 300.

That is to say, a fan of 5 feet diameter with 120 revolutions per minute—*or* one of 3 feet diameter with 200 revolutions per minute; *or* one of 2 feet diameter, having 300 revolutions per minute—would each of them give the required ventilation.

CHAPTER V

RAINFALL AND SEWERAGE

To Calculate the Amount of Water-supply available from Rainfall.

(i.) Amount of roof-space.

It is only necessary to ascertain the area of horizontal surface covered by the roof, the slope of the roof not affecting the result. Whatever its slope, the roof simply catches the rain which would have fallen on the area of horizontal ground, now covered by the roof, if the roof had not been there.

(ii.) Average amount of rainfall per annum is about 30 inches.

(iii.) The loss by evaporation is about one-fifth of the total rainfall.

Example :

The roof-space in a town is 60 square feet per head, and the annual rainfall is 30 inches. Find the amount of water available per head per annum.

Roof-space = 60 square feet ; rainfall = 30 inches = $2\frac{1}{2}$ feet ;

∴ amount of water per head per annum $=60\times 2\frac{1}{2}=$ 150 cubic feet.

Deduct $\frac{150}{5}=30$ for evaporation;

∴ available water $=150-30=120$ cubic feet,

$=120\times 6{\cdot}23=748$ gallons.

So the amount of water available for each individual during the year is 748 gallons, or about 2 gallons per diem.

Example:

How much water will 1 inch of rain deliver on 1 acre of ground?

One inch (or $\frac{1}{12}$ foot) of rain over 1 square yard (or 9 square feet) will give

$$9\times\frac{1}{12}=\frac{3}{4}\text{ cubic foot of water;}$$

$$=\frac{3}{4}\times 6{\cdot}23=4{\cdot}6725\text{ gallons.}$$

One acre will give $4{,}840\times 4{\cdot}6725=22{,}615$ gallons,

$=22{,}615\times 10=226{,}150$ pounds;

$=\frac{226{,}150}{2{,}240}=101$ tons (nearly)

(or, approximately, '100 tons per inch per acre').

[1 acre = 4,840 square yards.
1 cubic foot = 6·23 gallons.
1 gallon = 10 pounds.
2,240 pounds = 1 ton.]

General Formula for Rainfall.

Let R = annual rainfall (in inches),
E = annual evaporation (in inches);

Then (R – E) inches, or $\frac{R-E}{12}$ feet = available rainfall.

Let A = number of acres of collecting-ground,
Or (A × 4,840 × 9) = number of square feet of ground.

$$\text{Total water} = (A \times 4{,}840 \times 9) \times \frac{R-E}{12} \text{ cubic feet}$$
$$= 3{,}630\ A \times (R-E) \text{ cubic feet,}$$
$$= 3{,}630\ A \times (R-E) \times 6{\cdot}23 \text{ gallons,}$$
$$= 22{,}615 \times A \times (R-E) \text{ gallons per annum,}$$
$$= \frac{22{,}615 \times A \times (R-E)}{365},$$
$$= 62\ A\ (R-E) \text{ gallons per diem.}$$

Hawksley's Formula for Storage.

Let D = number of days' supply to be stored;
F = mean annual rainfall in inches;

$$\text{Then } D = \frac{1{,}000}{\sqrt{F}}.$$

In this formula, F may be taken to be either $\frac{5}{6}$ of the mean annual yield of several years, or the average of the three driest consecutive years

Example:

A cottage occupied by five persons has a horizontal roof-space of 360 square feet, and the annual rainfall is 25 inches; find the amount in gallons of the available supply per head per annum, and state in cubic feet the size of a tank needful to satisfy Hawksley's formula.*

* D.P.H. Exam., Cambridge.

Roof-space = 360 sq. feet; rainfall = 25 inches $= \frac{25}{12}$ feet.

$\therefore$ amount of water per annum $= 360 \times \frac{25}{12} = 750$ cub. feet.

Deduct $\frac{750}{5} = 150$ for evaporation,

$\therefore$ available water $= 750 - 150 = 600$ cub. feet.

$= 600 \times 6{\cdot}23 = 3{,}738$ gallons,

or $\frac{3{,}738}{5} = 748$ gallons per head per annum.

By Hawksley's formula $\frac{1{,}000}{\sqrt{25}} = \frac{1{,}000}{5} = 200$ days' supply must be stored.

The available daily supply for the five persons is

$$\frac{600}{365} \text{ cub. feet,}$$

$\therefore$ 200 days' supply $= \frac{600 \times 200}{365} = 328$ cub. feet,

which is the size, therefore, of the necessary tank.

FLOW IN SEWERS.

Hydraulic Mean Depth.

Example :

What is meant by 'Hydraulic Mean Depth'? What is it in the case of a circular pipe running half-full? Show that it is the same when the pipe runs full.*

The Hydraulic Mean Depth (or Radius) in a sewer is the sectional area of the fluid, divided by the length of that portion of the circumference of the pipe which is in contact with the fluid; or, more briefly:

* D.P.H. Exam. (Roy. Coll. Phys. Surg.).

$$\text{H.M.D.} = \frac{\text{sectional area of fluid}}{\text{wetted perimeter}}.$$

(i.) In a sewer **running full**, the sectional area of the fluid will be the same as that of the pipe, and the wetted perimeter will obviously be identical with the circumference;

$$\therefore \text{H.M.D.} = \frac{\pi r^2}{2\pi r} = \frac{r}{2} = \frac{d}{4} \text{ (p. 178).}$$

(ii.) In a sewer **running half-full**, the sectional area of the fluid will be exactly half that of the pipe, and the wetted perimeter will similarly be one-half the circumference;

$$\therefore \text{H.M.D.} = \frac{\frac{\pi r^2}{2}}{\frac{2\pi r}{2}} = \frac{\pi r^2}{2\pi r} = \frac{r}{2} = \frac{d}{4}.$$

That is, in a circular pipe running full or half-full, the H.M.D. is one-half the radius, or one-fourth the diameter.

Eytelwein's Formula.

This is founded upon the observation that the mean velocity per second in sewers is very nearly nine-tenths of a mean proportion between the H.M.D. and the fall in feet in 2 miles.

If D = hydraulic mean depth, and F = the fall in feet per mile, then 2F = fall in feet in 2 miles. And if x = mean proportion between these two quantities, then—

$$D : x :: x : 2F,$$
$$x^2 = 2DF, \text{ and}$$
$$x = \sqrt{2DF},$$

and nine-tenths of this will be $\frac{9 \times \sqrt{2DF}}{10}$.

Therefore, mean velocity per second $= \frac{9 \times \sqrt{2DF}}{10}$, and

mean velocity per minute $= \frac{9 \times \sqrt{2DF}}{10} \times 60 = 54\sqrt{2DF}$.

Eytelwein's formula gives—

$$V = 55\sqrt{2DF}$$

where V = velocity in feet per minute,
D = hydraulic mean depth in feet,
F = fall in feet per mile.

Having given Two of the Following—(*a*) Velocity of Flow; (*b*) Diameter of Sewer; (*c*) Gradient—to Find the Third.

(i.) A circular house-drain, running full, has a diameter of 4 inches; the gradient is 64 feet to the mile. What is the velocity of the discharge?

Making use of the above formula,

$$D = \frac{\text{diameter}}{4} = 1 \text{ inch} = \tfrac{1}{12} \text{ foot};$$

$$F = 64.$$

$$\therefore V = 55\sqrt{2 \times \tfrac{1}{12} \times 64} = 55\sqrt{10{\cdot}6} = 55 \times 3{\cdot}27$$
$$= 180 \text{ feet per minute,}$$
$$= 3 \text{ feet per second.}$$

(ii.) Calculate the gradient of a 12-inch sewer required to maintain a velocity of 3 linear feet per second.*

Diameter = 12 inches = 1 foot;
$\therefore$ D = $\frac{1}{4}$ foot.

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* D.P.H. Exam., Cambridge.

Velocity = 3 feet per second = 180 feet per minute;

$$\therefore\ 55\sqrt{2DF}=180,$$

$$55\sqrt{2.\tfrac{1}{4}.F}=180;$$

$$\therefore\ \sqrt{\frac{F}{2}}=\frac{180}{55}=\frac{36}{11},$$

$$\frac{F}{2}=\left(\frac{36}{11}\right)^2=\frac{1,296}{121},$$

$$F=\frac{1,296\times 2}{121}=21{\cdot}4;$$

$$\text{and}\quad \frac{21{\cdot}4}{5,280}\doteq\frac{1}{246}.$$

∴ Gradient required = 1 in 246, or a fall of 21·4 feet per mile.

(iii.) A circular drain is laid with a fall of 64 feet per mile. What should be its diameter in order that, when running full, it may discharge at the rate of 3 feet per second?

Let d = diameter required (in feet).

F = 64, and V = 3 feet per second = 180 feet per minute.

$$\text{Hydraulic mean depth}=\frac{\text{diameter}}{4};$$

$$\text{therefore } D=\frac{d}{4}.$$

$$V=55\sqrt{2DF}=55\sqrt{2\times\frac{d}{4}\times 64};$$

$$\therefore\ 180=55\sqrt{2\times\frac{d}{4}\times 64}=55\sqrt{32d};$$

$$\therefore\ \sqrt{32d}=\frac{180}{55}=\frac{36}{11};\ \therefore\ 32d=\left(\frac{36}{11}\right)^2=\frac{1296}{121},$$

$$\text{and } d=\frac{1296}{121\times 32}\text{ feet}=\frac{1296\times 12}{121\times 32}\doteq 4\text{ inches.}$$

Knowing the velocity and the sectional area, the discharge from the sewer can be calculated.

Example:

How much sewage would be discharged in 3 hours by a sewer, circular in shape, with a diameter of 18 inches and a fall of 1 in 180, running half-full?*

$$\text{Diameter} = 18 \text{ inches} = \frac{3}{2} \text{ feet};$$

$$\therefore D = \frac{3}{2} \times \frac{1}{4} = \frac{3}{8} \text{ foot},$$

$$\text{Fall} = 1 \text{ foot in } 180 \text{ feet} = \frac{5280}{180} \text{ feet in } 5{,}280 \text{ feet},$$

$$= 29{\cdot}3 \text{ feet per mile};$$

$$\therefore F = 29{\cdot}3.$$

$$V = 55\sqrt{2DF} = 55\sqrt{2 \times \tfrac{3}{8} \times 29{\cdot}3} = 55\sqrt{22} = 55 \times 4{\cdot}7$$

$$= 258{\cdot}5 \text{ feet per minute.}$$

$O = V \times S$ (p. 45).

$$S = \left(\frac{3}{2}\right)^2 \times 0{\cdot}7854 \text{ (p. 178)} = 1{\cdot}77 \text{ square feet};$$

$$\therefore O = 258{\cdot}5 \times 1{\cdot}77 = 4{,}575 \text{ cubic feet per minute,}$$

$$= 274{,}524 \text{ cubic feet per hour,}$$

$$= 823{,}572 \text{ cubic feet in 3 hours};$$

or, $823{,}572 \times 6{\cdot}23 = 5{,}130{,}853$ gallons in 3 hours.

And half this amount—viz., 2,565,427 gallons—will represent the discharge in 3 hours, when running half-full.

Example:

Over an area of 1,000 acres a rainfall of $\frac{1}{6}$ inch per hour occurs. What must be the diameter of a drain to carry this away, the gradient being 10 feet per mile?

The diameter can be ascertained from the sectional

* D.P.H. Exam. (Roy. Coll. Phys. Surg.).

area, and the sectional area may be found if the velocity and outflow are known, since $S=\frac{O}{V}$ (p. 45); therefore:

(i.) Find the outflow.

$$1{,}000 \text{ acres}=1{,}000\times 4{,}840 \text{ square yards},$$
$$=1{,}000\times 4{,}840\times 9=43{,}560{,}000 \text{ square feet};$$
$$\text{and } \frac{1}{6} \text{ inch}=\frac{1}{72} \text{ foot};$$

$\therefore$ Amount of rain falling $=43{,}560{,}000\times\frac{1}{72}=605{,}000$ cubic feet per hour;

or, outflow $=10{,}083$ cubic feet per minute.

(ii.) Find the velocity of flow.

Let $d=$ diameter of drain required (in feet);

$$V=55\sqrt{2DF};\ D=\frac{d}{4} \text{ (p. 71)};\ F=10;$$
$$\therefore V=55\sqrt{2\times\frac{d}{4}\times 10}=55\sqrt{5d}=55\times 2{\cdot}236\sqrt{d}$$
$$=122{\cdot}98\sqrt{d} \text{ feet per minute.}$$

(iii.) Find sectional area of drain.

$$S=\frac{O}{V}=\frac{10083}{122{\cdot}98\sqrt{d}}=\frac{81{\cdot}98}{\sqrt{d}} \text{ square feet.}$$

(iv.) Find the diameter.

$$S=d^2\times 0{\cdot}7854 \text{ (p. 178)};$$
$$\therefore d^2=\frac{S}{0{\cdot}7854}=\frac{81{\cdot}98}{0{\cdot}7854\times\sqrt{d}};$$
$$\therefore d^2\times\sqrt{d}=\frac{81{\cdot}98}{0{\cdot}7854}=104{\cdot}39;$$
$$\text{that is, } d^{\frac{5}{2}}=104{\cdot}39;$$
$$\therefore \frac{5}{2}\log d=\log 104{\cdot}39=2{\cdot}0186589;$$
$$\therefore \log d=\frac{2\times 2{\cdot}0186589}{5}=0{\cdot}8074636=\log 6{\cdot}4;$$
$$\therefore d=6{\cdot}4 \text{ feet.}$$

[No deduction has been made for loss by evaporation.]

Example:

How many gallons of water per minute would be discharged from a drain-pipe having a diameter of 2 inches and a slope of '1 in 10,' using for the calculation the formula—

$$V = 140\sqrt[2]{HS} - 11\sqrt[3]{HS},$$

where V = the number of linear feet issuing per second,
H = the hydraulic mean depth,
S = sine of the slope.

(Tables of logarithms are provided.)*

Draw any horizontal line AB, such that the length AB = 10 units. From the point B draw BC perpendicular to AB, and of a length equal to 1 unit; join AC. Then AC represents the slope of the drain (1 in 10), and the sine of the slope is the sine of the angle BAC.

The length of the line AC is $\sqrt{(AB)^2 + (BC)^2}$. (Euclid I. 47.)

$$= \sqrt{(10)^2 + (1)^2}$$
$$= \sqrt{101}$$
$$= 10{\cdot}05.$$

The sine of an angle is given by the trigonometrical ratio $\dfrac{\text{perpendicular}}{\text{hypotenuse}}$;

$$\therefore \text{ Sine } BAC = \frac{BC}{AC} = \frac{1}{10{\cdot}05} = S.$$

The diameter of the drain = 2 inches = $\frac{1}{6}$ foot; therefore the hydraulic mean depth = $\frac{1}{4} \times \frac{1}{6} = \frac{1}{24}$ feet.

$$\therefore HS = \frac{1}{24} \times \frac{1}{10{\cdot}05} = \frac{1}{241{\cdot}2} = 0{\cdot}004146;$$

$$\therefore V = 140\sqrt[2]{0{\cdot}004146} - 11\sqrt[3]{0{\cdot}004146}.$$

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* D.P.H. Exam., Cambridge.

Now $\log. 140\sqrt[2]{0{\cdot}004146} = \log. 140 + \frac{1}{2} \log. 0{\cdot}004146$;

$$= 2{\cdot}1461280 + \frac{\bar{3}{\cdot}6176281}{2};$$

$$= 2{\cdot}1461280 + \bar{2}{\cdot}8088140;$$

$$= 0{\cdot}9549420;$$

$$= \log. 9{\cdot}0145;$$

$$\therefore 140\sqrt[2]{0{\cdot}004146} = 9{\cdot}0145.$$

Similarly, it may be shown that—

$$11\sqrt[3]{0{\cdot}004146} = 1{\cdot}7671;$$

Therefore $V = 9{\cdot}0145 - 1{\cdot}7671 = 7{\cdot}2474$ linear feet per second $= 434{\cdot}8$ linear feet per minute.

Diameter of drain $= \frac{1}{6}$ foot;

$$\therefore \text{Sect. area} = (\tfrac{1}{6})^2 \times 0{\cdot}7854 \text{ (p. 178)},$$

$$= \frac{0{\cdot}7854}{36} \text{ sq. feet.}$$

$$\therefore \text{Outflow} = 434{\cdot}8 \times \frac{0{\cdot}7854}{36} \text{ cub. feet per minute (p. 45)},$$

$$\text{or } \frac{434{\cdot}8 \times 0{\cdot}7854 \times 6{\cdot}23}{36} \text{ gallons per minute.}$$

This multiplication and division should be performed by logarithms as follows:

Log. outflow—

$$= \log. 434{\cdot}8 + \log. 0{\cdot}7854 + \log. 6{\cdot}23 - \log. 36,$$

$$= 2{\cdot}6382895 + \bar{1}{\cdot}8950899 + 0{\cdot}7944880 - 1{\cdot}5563025,$$

$$= 1{\cdot}7715649,$$

$$= \log. 59{\cdot}097;$$

$$\therefore \text{Outflow} = 59 \text{ gallons per minute.}$$

The Relationship between the Diameter, Velocity, and Gradient of Sewers.

Example:

How does the fall of a sewer affect the velocity of flow in it?*

If V = present velocity, and
v = the velocity to be found,

$$\text{then } V = 55\sqrt{2DF}, \text{ and } v = 55\sqrt{2D'F'};$$
$$\text{and } V : v :: 55\sqrt{2DF} : 55\sqrt{2D'F'};$$
$$:: \sqrt{2DF} : \sqrt{2D'F'};$$
$$\therefore V^2 : v^2 :: 2DF : 2D'F';$$
$$:: DF : D'F';$$

but since the diameter does not vary, $D = D'$,

$$\therefore V^2 : v^2 :: F : F'$$

that is—

(i.) Where the diameter is constant, the fall varies as the square of the velocity.

Similarly, it may be shown that—

(ii.) Where the velocity is constant, the fall varies inversely as the diameter, or

$$F : F' :: d' : d;$$

and—

(iii.) Where the fall is constant, the diameter varies as the square of the velocity, or

$$V^2 : v^2 :: d : d'.$$

Example:

Show by the formula $V = 55\sqrt{H2F}$ that sewer-pipes of the following diameters and gradients have each the same velocity of outflow:

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* D.P.H. Exam. (Roy. Coll. Phys. Surg.).

(*a*) Diameter 5 feet, fall 4 feet per mile.
(*b*) Diameter 2 feet, fall 10 feet per mile.
(*c*) Diameter 1 foot, fall 20 feet per mile.*

Changing H (hydraulic mean depth) in the above formula to D, and so using the same notation as has been already adopted in this chapter, the formula becomes $V=55\sqrt{2DF}$.

If a sewer of 5 feet diameter has a fall of 4 feet per mile, what must the fall be in the case of a sewer 2 feet in diameter to ensure the same velocity?

$$F : F' :: d' : d;$$
$$\therefore 4 : F' :: 2 : 5;$$
$$\therefore 2F'=5\times 4=20;$$
$$\therefore F'=10.$$

That is, to obtain in (*b*) the same velocity as in (*a*), the gradient in (*b*) must be one of 10 feet per mile, which proves the proposition in the case of (*b*).

Similarly for (*c*):

$$F : F' :: d' : d,$$
$$4 : F' :: 1 : 5,$$
$$F'=5\times 4=20,$$

which proves (*c*).

Example:

Estimate the relative volumes of sewage discharged from the three sewers whose dimensions and gradients are given in the previous example.

$$\text{Sect. area}=d^2\times 0{\cdot}7854 \text{ (p. 178)};$$
$$\therefore \text{Sect. area of } (a)=(5)^2\times 0{\cdot}7854;$$
$$\text{,, ,, } (b)=(2)^2\times 0{\cdot}7854;$$
$$\text{,, ,, } (c)=(1)^2\times 0{\cdot}7854.$$

* D.P.H. Exam., Cambridge.

Thus, the sectional areas vary as the squares of the diameters. The gradients are such as to produce the same velocity in each,

$$\text{and } V = \frac{O}{S};$$

therefore, since V is constant, the volume of sewage discharged (O) must vary directly as the sectional area (S), and therefore directly as the square of the diameter of the sewer.

And since the squares of the diameters of (*a*), (*b*), and (*c*) are in the proportion of 25, 4, and 1 respectively, the volumes of sewage discharged will be in the same ratio; that is, (*a*) will discharge 25 times, and (*b*) 4 times the volume that (*c*) will in a given time, and with equal velocity.

EXCRETA.

Estimation of the Amount of Nitrogen and Ammonia in Excreta.

(i.) *An average adult man* passes daily:

4 ounces of fæces and 50 ounces of urine.

1 fluid ounce water (sp. gr. 1000) weighs 1 ounce $= 437{\cdot}5$ grains;

$\therefore$ 1 fluid ounce urine (sp. gr. 1020) $= \frac{437{\cdot}5 \times 1020}{1000}$

$= 446{\cdot}25$ grains;

$\therefore$ 50 fluid ounces $= 446{\cdot}25 \times 50 = 22{,}312$ grains.

Urine contains 4·2 per cent. solids;

$$\therefore \text{solids} = \frac{22312 \times 4{\cdot}2}{100} = 937{\cdot}1 \text{ grains.}$$

Of the solids, 54 per cent. is urea;

$$\therefore \text{urea} = \frac{937{\cdot}1 \times 54}{100} = 506 \text{ grains.}$$

Urea—$CO(NH_2)_2$—contains 28 parts N in 60 parts :

$$\therefore N = \frac{506 \times 28}{60} = 236 \text{ grains.}$$

14 grains N are contained in 17 grains NH_3 ;

$$\therefore 236 \text{ grains } N = \frac{236 \times 17}{14} = 287 \text{ grains } NH_3.$$

Fæces contains 23·4 per cent. solids ;

$\therefore$ 4 ounces (or 1,750 grains) contain $\frac{1750 \times 23 \cdot 4}{100} =$ 409·5 grains solids.

Of the solids, about 16 per cent. is N ;

$$\therefore N = \frac{409 \cdot 5 \times 16}{100} = 65 \cdot 5 \text{ grains ;}$$

$$\text{and } NH_3 = \frac{65 \cdot 5 \times 17}{14} = 79 \cdot 5 \text{ grains.}$$

Therefore, the daily excreta of an *adult man* contain

Total $N = 236 + 65 = 301$ grains.
Total $NH_3 = 287 + 79 = 366$ grains.

(ii.) *In a mixed community* (including women and children) the daily quantity per head averages :

2·5 ounces fæces, and 40 ounces urine ;

and the amount of N contained therein can be estimated in a precisely similar manner and with the following results :

40 ounces urine contain 189 grains N, or 229 grains NH_3 ;
2·5 ounces fæces contain 41 grains N, or 50 grains NH_3.

Therefore, the daily excreta per head of a *mixed community* contain :

Total N = 189 + 41 = 230 grains.
Total NH_3 = 229 + 50 = 279 grains.

[These quantities, dealing with solids only, are water-free.]

Example:

How much solid and liquid excreta are passed by an average adult man per diem, and how much water-free solids does the amount represent? How much per annum would a mixed community of 50,000 persons pass, and how much ammonia would be contained in the total bulk? *

An average adult man passes daily:

4 ounces fæces and 50 ounces urine.

Fæces contain 23·4 per cent. solids;

$$\therefore \text{ 4 ounces contain } \frac{4 \times 23{\cdot}4}{100} = 0{\cdot}936 \text{ ounces solids.}$$

Urine contains 4·2 per cent. solids;

$$\therefore \text{ 50 ounces contain } \frac{50 \times 4{\cdot}2}{100} = 2{\cdot}1 \text{ ounces solids;}$$

$$\therefore \text{ total water-free solids} = 0{\cdot}936 + 2{\cdot}1 = 3{\cdot}036 \text{ ounces.}$$

In a mixed community the quantities per head per diem average:

2·5 ounces fæces, and 40 ounces urine.

Therefore, in a year, the fæces of 50,000 persons would amount to:

$$2{\cdot}5 \times 365 \times 50{,}000 \text{ ounces;}$$

$$\text{or, } \frac{2{\cdot}5 \times 365 \times 50000}{16 \times 2240} = 1{,}273 \text{ tons;}$$

and the urine would amount to:

$$\frac{40 \times 365 \times 50000}{16 \times 2240} = 20{,}368 \text{ tons.}$$

* D.P.H. Exam., Cambridge.

Total excreta per annum =

$$1{,}273 + 20{,}368 = 21{,}641 \text{ tons.}$$

[These quantities are not water-free.]

The average daily excreta of a mixed community per head has been seen (p. 82) to contain 279 grains of NH_3; therefore, in a year, the excreta of 50,000 persons would contain:

$$279 \times 365 \times 50{,}000 \text{ grains;}$$

$$\text{or, } \frac{279 \times 365 \times 50{,}000}{437{\cdot}5 \times 16 \times 2{,}240} = 325 \text{ tons } NH_3.$$

If, in the first part of this example, the solids, instead of being water-free, had contained 25 per cent. of moisture, then the remaining 75 per cent. would represent the solids; that is:

$$3{\cdot}036 \text{ ounces} = 75 \text{ per cent. of the whole,}$$

$$\text{and } \frac{3{\cdot}036 \times 100}{75} = 4{\cdot}048 \text{ ounces} = \text{total weight.}$$

CHAPTER VI

ENERGY, EXERCISE, AND DIET

THE fact that any agent is capable of doing work is usually expressed by saying that it possesses energy, and the quantity of energy it possesses is measured by the amount of work it can do. The 'unit of work' is generally taken to be the quantity of work which is done in lifting 1 pound through a height of 1 foot, and this quantity of work is called 1 'foot-pound.' The product of the weight lifted (expressed in pounds), into the height through which it is lifted (expressed in feet), gives the amount of work done (in foot-pounds).

Thus, a weight of 20 pounds lifted through a distance of 1 foot = 20 foot-pounds; or, a weight of 1 pound lifted through a distance of 20 feet = 20 foot-pounds.

In the same way, the 'work done' by a man during exercise can be ascertained.

If W = his weight (in pounds) and D = the vertical height (in feet) to which he lifts his body, then the work done = W × D foot-pounds; or if D = height in *miles*, then 5,280D = height in *feet*, and work done

$$= W \times 5{,}280D \text{ foot-pounds, or } \frac{W \times 5280D}{2240} \text{ foot-}\textit{tons}.$$

So far the lifting of the man's weight vertically upwards has only been considered. What will the work amount to if, instead, he propel his weight horizontally along level ground? A great deal will depend upon the velocity with which he walks. It has been calculated that at an ordinary rate of three miles per hour, a man, walking along level ground, does work equivalent to raising his own weight, vertically, through $\frac{1}{20}$ the distance travelled; or, what is the same thing, raises $\frac{1}{20}$ of his weight through the whole distance travelled.

It has been seen that raising his whole weight through the distance D miles requires an expenditure of $\frac{W \times 5280D}{2240}$ foot-tons; therefore, to raise $\frac{1}{20}$ of his weight through the distance D will only require an expenditure of $\frac{W \times 5280D}{2240} \times \frac{1}{20}$ foot-tons; which number, therefore, gives the work done in walking a distance of D miles on level ground, at the rate of three miles per hour.

W has been taken to represent the total weight—*i.e.*, weight of body + weight carried.

If W = weight of body, and W′ = weight carried, then the formula becomes

$$\frac{(W + W') \times 5280D}{2240} \times \frac{1}{20} \text{ foot-tons.}$$

The fraction '$\frac{1}{20}$' is spoken of as the 'coefficient of resistance' (or traction), and varies with the rate of walking. At three miles per hour, on level ground, it is equivalent to $\frac{1}{20\cdot59}$ $\left(\text{approximately } \frac{1}{20}\right)$, at four miles $= \frac{1}{16\cdot75}$, and at five miles $= \frac{1}{14\cdot10}$.

If C = coefficient of resistance, then the formula may be written

$$\frac{(W+W') \times 5280D}{2240} \times C \text{ foot-tons.}$$

Where W = man's weight (in pounds).
W′ = weight carried ,,
D = distance walked (in miles).
C = coefficient of resistance.

EXERCISE.

Example:

A bricklayer's labourer weighs 150 lbs. and carries a load of bricks—40 lbs.—up a perpendicular ladder 30 feet high 100 times a day. Calculate how much external work he does daily, and what will it be equal to in miles walked on the flat at the rate of 3 miles an hour?*

$$150 + 40 = 190 \text{ lbs.} = \frac{190}{2,240} \text{ ton.}$$

This weight has to be lifted through a height of 30 feet 100 times a day, or through a total height of 3,000 feet;

$$\therefore \text{ total work} = \frac{190}{2,240} \times 3,000 = 254 \text{ foot-tons daily.}$$

Find the equivalent of this in miles walked on the flat at the rate of 3 miles an hour:

$$\frac{(W+W') \times 5,280D}{2,240} \times C;$$

where $W + W' = 190$,
D = distance walked in miles,
$C = \frac{1}{20}$;
total work = 254 foot-tons;

* D.P.H. Exam., Cambridge.

$$\therefore \quad \frac{190 \times 5{,}280D}{2{,}240} \times \tfrac{1}{20} = 254,$$

$$\text{and } D = \frac{254 \times 20 \times 2{,}240}{190 \times 5{,}280} = 11{\cdot}3 \text{ miles.}$$

For the perpendicular ladder substitute a plank 30 feet long, with a gradient of 1 in 12.

Total distance walked = 3,000 feet.

If the plank rises 1 foot in 12 feet it will rise

$$\frac{3{,}000}{12} = 250 \text{ feet}$$

in the total distance walked. Therefore, in addition to walking 3,000 feet on level ground, he has to raise his whole weight through a height of 250 feet. The energy expended in this portion of work will be

$$\frac{190}{2{,}240} \times 250 = 21{\cdot}2 \text{ foot-tons ;}$$

and in traversing the distance on level ground at the rate of 3 miles an hour, the work done will be

$$\frac{190}{2{,}240} \times 3{,}000 \times \tfrac{1}{20} = 12{\cdot}7 \text{ foot-tons.}$$

$$\therefore \text{ Total work} = 21{\cdot}2 + 12{\cdot}7 = 34 \text{ foot-tons.}$$

[NOTE.—The distance travelled on level ground is equivalent to raising $\frac{1}{20}$ of the weight through the whole distance. Whilst the upward distance is equivalent to raising the *whole* weight through the vertical height ascended.

An 'ordinary day's work' is generally taken to be 300 foot-tons ; and this amount of energy would be expended in walking 14·2 miles on level ground at the rate of 3 miles per hour.]

METHODS OF CALCULATING DIETS.

Method 1:

The total amount of carbon and nitrogen necessary for each individual per diem must be known; a table also is required giving the amount of C and N in a fixed quantity (*e.g.*, 1 ounce) of the different foods. The N should be to the C in the ratio of 1 : 15; and for ordinary work, 300 grains of N and 4,500 grains of C may be taken as a standard.

Example:

Find the amount of bread and milk sufficient for the diet of a man at ordinary work.

Let x = amount of bread (in ounces).
y = ,, ,, milk

From a table it is found that—

1 ounce of bread contains 116 grains C and 5·5 grains N.
1 ounce of milk contains 30 grains C and 2·8 grains N.

Therefore—

x ounces bread contain $116x$ grains C and $5{\cdot}5x$ grains N.
y ounces milk contain $30y$ grains C and $2{\cdot}8y$ grains N;
and total C = 4,500; total N = 300.

$$\therefore \text{ total carbon} = 116x + 30y = 4{,}500 \ldots\ldots \text{(i.)}.$$
$$\text{total nitrogen} = 5{\cdot}5x + 2{\cdot}8y = 300 \ldots\ldots \text{(ii.)}.$$

From these two equations find the values of x and y.

Eliminate y by multiplying each equation by the coefficient of y in the other equation—that is, by multiplying (i.) by 2·8 and (ii.) by 30. Thus:

$$324{\cdot}8x + 84y = 12{,}600 \ldots\ldots \text{(iii.)}.$$
$$165x + 84y = 9{,}000 \ldots\ldots\ldots \text{(iv.)}.$$

Subtract (iv.) from (iii.), and the difference is—

$$159{\cdot}8x = 3{,}600;$$
$$\therefore x = \frac{3600}{159{\cdot}8} = 22{\cdot}5.$$

Substitute this value of x in (ii.), thus :

$$(5{\cdot}5 \times 22{\cdot}5) + 2{\cdot}8y = 300;$$
$$123{\cdot}75 + 2{\cdot}8y = 300;$$
$$2{\cdot}8y = 300 - 123{\cdot}75 = 176{\cdot}25;$$
$$\therefore y = 63.$$

That is, $22\frac{1}{2}$ ounces of bread and 63 ounces of milk will supply the necessary amount of carbon and nitrogen.

This problem may also be solved graphically as follows :

$$116x + 30y = 4{,}500 \ldots\ldots \text{ (i.).}$$
$$5{\cdot}5x + 2{\cdot}8y = 300 \ldots\ldots\ldots \text{ (ii.).}$$

Since these equations are of the first degree, their graphs will be straight lines.

In (i.) if $x=0$, then $y=150$
and if $y=0$, then $x=39$,

the points (0, 150) and (39, 0) will thus be on the graph, and the straight line drawn through these points will be the graph of equation (i.).

Similarly for (ii.), if $x=0$, then $y=107$,
and if $y=0$, then $x=54{\cdot}5$.

Therefore a straight line drawn through the points (0, 107) and (54·5, 0) will be the graph of equation (ii.).

These lines will be found to intersect at the point (22·5, 63) which gives the solution of the problem.

Example :

How much cane-sugar and dry albumen are required in a mixed dietary to furnish 30 grammes of nitrogen and 350 grammes of carbon ?*

Let x = number of grammes of albumen.
y = number of grammes of cane-sugar.

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* D.P.H. Exam. (Roy. Coll. Phys. Surg.).

The formula for cane-sugar ($C_{12}H_{22}O_{11}$) shows that it contains 144 parts by weight of C in 342 parts, or $\frac{144 \times 100}{342} = 42$ per cent. (see example on p. 109).

Proteids—*e.g.*, albumen—contain approximately 53 per cent. C and 15 per cent. N, that is :

100 grammes albumen contain 53 grammes C ;

$\therefore x$ grammes albumen contain $\frac{53 \times x}{100}$ grammes C.

100 grammes cane-sugar contain 42 grammes C ;

$\therefore y$ grammes cane-sugar contain $\frac{42 \times y}{100}$ grammes C.

Total C must be 350 grammes ;

$$\therefore \frac{53x}{100} + \frac{42y}{100} = 350;$$

$$\text{or, } 53x + 42y = 35{,}000 \ldots\ldots \text{(i.)}.$$

100 grammes albumen contain 15 grammes N ;

$\therefore x$ grammes albumen contain $\frac{15 \times x}{100}$ grammes N.

Total N must be 30 grammes ;

$$\therefore \frac{15x}{100} = 30;$$

$$\text{or, } 15x = 3{,}000 \ldots\ldots \text{(ii.)}.$$

Equation (ii.) shows that $x = \frac{3000}{15} = 200$.

Substitute this value of x in (i.), thus :

$$(53 \times 200) + 42y = 35{,}000;$$
$$10{,}600 + 42y = 35{,}000;$$
$$42y = 35{,}000 - 10{,}600 = 24{,}400;$$
$$\therefore y = 581.$$

200 grammes of albumen and 581 grammes of cane-sugar are therefore required.

Method 2 :

A 'standard diet' is required, showing the total amount of each variety of food which is necessary, and also a table, giving the percentage composition of different foods.

A Report—drawn up by a Committee of the Royal Society, at the request of the Board of Trade—was issued in 1917, which stated that 'a nation, for the most part engaged in active work,' requires per man per diem :

100 grms. proteids,
100 grms. fat, and
500 grms. carbohydrates.

(These figures will be adopted here as 'the standard diet.')

Example:

Find the amount of bread, cheese, and butter sufficient for the diet of a man in active work.

Let x = amount of bread (in grammes).
y = „ cheese „
z = „ butter „

From a table (Parkes and Kenwood) it is found that

	Proteids, per cent.	Fats, per cent.	Carbohydrates, per cent.
Bread contains	8·0	0·5	50
Cheese „	28·0	23·0	1
Butter „	1·5	83·5	1

Therefore—

	Proteids.	Fats.	Carbohydrates.	
x grms. bread contain	$\frac{8x}{100}$	$\frac{0{\cdot}5x}{100}$	$\frac{50x}{100}$	grms.
y grms. cheese „	$\frac{28y}{100}$	$\frac{23y}{100}$	$\frac{y}{100}$	„
z grms. butter „	$\frac{1{\cdot}5z}{100}$	$\frac{83{\cdot}5z}{100}$	$\frac{z}{100}$	„

But the proteids must be equal to a total of 100 grms., the fats 100 grms., and the carbohydrates 500 grms.

$$\therefore \text{ Proteids} = \frac{8x}{100} + \frac{28y}{100} + \frac{1{\cdot}5z}{100} = 100;$$

$$\text{Fats} = \frac{0{\cdot}5x}{100} + \frac{23y}{100} + \frac{83{\cdot}5z}{100} = 100;$$

$$\text{Carbohydrates} = \frac{50x}{100} + \frac{y}{100} + \frac{z}{100} = 500.$$

Clear of fractions by multiplying each of these three equations by 100, thus:

$$8x + 28y + 1{\cdot}5z = 10{,}000 \ldots\ldots\ldots \text{(i.)}.$$
$$0{\cdot}5x + 23y + 83{\cdot}5z = 10{,}000 \ldots\ldots \text{(ii.)}.$$
$$50x + y + z = 50{,}000 \ldots\ldots\ldots\ldots \text{(iii.)}.$$

There are here three equations, with three unknown quantities, and the first stage in the solution consists in reducing these to two equations, with two unknowns.

First eliminate x from (i.) and (ii.) by multiplying (ii.) by 8, and (i.) by 0·5, thus:

$$4x + 184y + 668z = 80{,}000 \ldots\ldots \text{(iv.)}.$$
$$4x + 14y + 0{\cdot}75z = 5{,}000 \ldots\ldots\ldots \text{(v.)}.$$

Subtract (v.) from (iv.), and the difference is

$$170y + 667{\cdot}25z = 75{,}000 \ldots\ldots\ldots \text{(vi.)}.$$

Secondly, eliminate x from (i.) and (iii.), by multiplying (i.) by 50, and (iii.) by 8, thus:

$$400x + 1{,}400y + 75z = 500{,}000 \ldots\ldots \text{(vii.)}$$
$$400x + 8y + 8z = 400{,}000 \ldots\ldots\ldots\ldots \text{(viii.)}$$

Subtract (viii.) from (vii.), and the difference is

$$1{,}392y + 67z = 100{,}000 \ldots\ldots\ldots\ldots \text{(ix.)}.$$

Thus are obtained two equations, containing two unknown quantities only—viz.,

$$170y + 667{\cdot}25z = 75{,}000 \quad \ldots\ldots \text{(vi.)}.$$
$$1{,}392y + 67z = 100{,}000 \quad \ldots\ldots \text{(ix.)}.$$

Next these two equations with two unknown quantities must be reduced to one equation with one unknown.

Eliminate y by multiplying (vi.) by 1,392, and (ix.) by 170, thus:

$$236{,}640y + 928{,}812z = 104{,}400{,}000 \quad \ldots\ldots \text{(x.)}.$$
$$236{,}640y + 113{,}90z = 17{,}000{,}000 \quad \ldots\ldots \text{(xi.)}.$$

Subtract (xi.) from (x.), and the difference is

$$917{,}422z = 87{,}400{,}000;$$
$$\therefore\ z = \frac{87{,}400{,}000}{917{,}422} = 95{\cdot}2$$

Substitute this value of z in (ix.), thus:

$$1{,}392y + (67 \times 95{\cdot}2) = 100{,}000;$$
$$1{,}392y = 100{,}000 - 6{,}378{\cdot}4;$$
$$= 93{,}621{\cdot}6;$$
$$y = \frac{93{,}621{\cdot}6}{1{,}392} = 68.$$

Substitute the found values of y and z in (iii.), thus:

$$50x + 68 + 95{\cdot}2 = 50{,}000;$$
$$50x = 49{,}836{\cdot}8;$$
$$x = \frac{49{,}836{\cdot}8}{50} = 997.$$

That is, 997 grms. (35 oz.) of bread, 68 grms. (2·4 oz.) of cheese, and 95·2 grms. (3·3 oz.) of butter will supply the requisite amount of proteids, fats, and carbohydrates.

What is the energy available from 'the standard diet'?

When oxidised in the body, the energy developed by

1 grm. water-free proteid = 6·1 foot-tons.
1 grm. „ „ fat = 13·33 „ „
1 grm. „ „ carbohydrate = 4·76 foot-tons.

Therefore—

100 grms. proteids yield $100 \times 6 \cdot 1$	$=$ 610	foot-tons
100 grms. fats yield $100 \times 13 \cdot 33$	$=$ 1,333	,,
500 grms. carbohydrates yield $500 \times 4 \cdot 76$	$=$ 2,380	,,
Total	4,323	,,

So it is seen that the standard diet for ordinary active work yields rather more than 4,000 foot-tons of potential energy. Of these 4,000 foot-tons, 300 foot-tons, on an average, are expended in external mechanical work (p. 87), and 260 foot-tons on internal bodily work (being utilised by the heart and respiratory organs, etc.).

Thus the total for internal and external work $= 300 + 260 = 560$, which is less than $\frac{1}{7}$ of the total energy available from the food.

THE FUEL-VALUE OF FOOD.

The amount of heat required to raise 1 litre (*i.e.*, 1 kilogram) of water through 1° C. is termed a *calorie*.

The heat value of 1 grm. proteid	$= 4 \cdot 1$	calories.
,, ,, ,, carbohydrate	$= 4 \cdot 1$	,,
,, ,, ,, fat	$= 9 \cdot 3$	,,

What is the heat value of the standard diet for ordinary work?

100 grms. proteid $= 100 \times 4 \cdot 1$	$=$ 410	calories.
100 grms. fat $= 100 \times 9 \cdot 3$	$=$ 930	,,
500 grms. carbohydrate $= 500 \times 4 \cdot 1$	$=$ 2,050	,,
Total	3,390	,,

That is, the standard diet yields approximately 3,400 calories.

Example:

Find the value of a calorie when the unit of weight is 1 lb., and that of temperature 1° F.

$100° C. = 180° F., \therefore 1° C. = 1{\cdot}8° F. \quad 1 kilo = 2{\cdot}2 lbs.$

$$1 \text{ cal rie} = 1 \text{ kilo raised } 1° C.$$
$$\therefore 1 \text{ calorie} = 2{\cdot}2 \text{ lbs. raised } 1{\cdot}8° F.$$
$$= 1 \text{ lb. raised } 2{\cdot}2 \times 1{\cdot}8 = 4° F.$$

Therefore a calorie may also be defined as the amount of heat required to raise 1 lb. of water through 4° F.

Example:

Calculate the heat-value of 1 lb. of butter.

Butter has the following percentage composition (p. 91): Proteids 1·5, Fats 83·5, Carbohydrates 1 per cent.

∴ Proteids	= 1·5 × 4·1 =	6·15	calories.
Fats	= 83·5 × 9·3 =	776·55	,,
Carbohydrates	= 1 × 4·1 =	4·1	,,
	Total	786·80	,,

That is, 100 grms. butter yield 786·8 calories ; therefore 1 lb. (or 453·6 grms.) of butter yields

$$\frac{786{\cdot}8 \times 453{\cdot}6}{100} = 3{,}569 \text{ calories.}$$

That is, the combustion of 1 lb. of butter would provide sufficient heat to raise 3,569 kilograms of water through 1° C., or 3,569 lbs. of water through 4° F.

CHAPTER VII

THE CONSTRUCTION OF A HOSPITAL WARD

SUPPOSING it is wished to construct a ward to accommodate twenty patients, allowing 1,000 cubic feet of space per head—that is to say, a ward whose cubic capacity is 20,000 cubic feet. The variations in its possible dimensions are endless—*e.g.*, the following would satisfy the conditions as to cubic capacity :

	Length.		*Width.*		*Height.*
1.	100 feet	...	20 feet	...	10 feet
2.	50 "	...	40 "	...	10 "
3.	40 "	...	25 "	...	20 "
4.	32 "	...	25 "	...	25 "
	etc.		etc.		etc.

But some of these would be useless for the accommodation of twenty patients—*e.g.*, if ten beds were placed on each side of a ward constructed as No. 4, each bed (3 feet wide) would only have $\frac{32}{10} = 3{\cdot}2$ feet of wall-space, and the enormous height (25 feet) is simply wasted space. Obviously, then, the total amount of cubic space required will not help us much ; and it may be remarked at the outset that it is not sufficient for the ward, *taken as a whole*, to satisfy (as the above examples do) the total gross conditions as to capacity, etc., but that it is necessary for each individual patient's portion of the ward to satisfy certain conditions. What these are will now be inquired into more fully.

Number of Patients:

The *number* of patients to be accommodated is of less importance than the consideration how *each* patient should be accommodated. The maximum number, however, should not exceed 30.

Conditions as to Height:

Anything above 14 or 15 feet should be neglected in calculating the necessary cubic space per head, since organic impurities tend to collect in the lower portions of the atmosphere, and excessive height will not counterbalance this, whereas increased space in other dimensions would dilute these.

Width:

The minimum width should be 24 feet ; allowing $6\frac{1}{2}$ feet for the length of each bed, two beds on opposite sides of the ward would take up 13 feet, leaving $24-13=11$ feet for the passage down the middle of the ward between the two rows.

Length:

No limits within reason. If we have a ward containing twenty patients, and it is wished to enlarge it into one accommodating thirty, it is obvious that the only way it could be done would be by adding to the length ; for, as has been already seen, increase in height should not be allowed to provide for the extra amount of cubic space required, and increase in width would only widen the passage down the middle of the ward between the two rows of beds without providing any further bed accommodation.

Floor-Space:

From what has been said as to height, it will be seen that dilution of respiratory impurities can only be thoroughly carried out by each patient having a certain amount of floor-space, and the minimum should be 100 square feet per head, and should not be less than $\frac{1}{12}$ of the cubic space.

Amount of Cubic Space:

For a general hospital, each patient should have a minimum of 1,200 cubic feet; if the air be renewed three times per hour, this would give 3,600 cubic feet of fresh air per head per hour.

Windows:

It is generally allowed that there should be 1 square foot of window to every 70 cubic feet of space; they should reach downwards to within $2\frac{1}{2}$ or 3 feet of the floor, and upwards to within 1 foot of the ceiling. Where a bed is placed between each window, the space between the windows should be at least 1 foot wider than the width of the bed; as the beds are usually 3 feet wide, the space between the windows must be at least 4 feet.

Beds:

Three feet should be the minimum distance between adjoining beds.

Let us now see how to apply these data in constructing a ward of the usual oblong form to accommodate, *e.g.*, twenty-eight ordinary medical patients.

Floor-space must be a minimum of $100 \times 28 =$ 2,800 square feet.

Cubic contents must be a minimum of $1,200 \times 28 = 33,600$ cubic feet.

It will be noticed that this exactly fulfils the condition that floor-space should be at least $\frac{1}{12}$ of the cubic space, since $\frac{33,600}{12} = 2,800$. Knowing the cubic space to be 33,600 cubic feet, and the floor-space 2,800 square feet, it will be seen that a height of $\frac{33,600}{2,800} = 12$ feet will satisfy the conditions so far. Having obtained the height and the floor-space, the latter must now be divided up into length and breadth. Taking the minimum permissible width—viz., 24 feet—we are left with $\frac{2,800}{24} = 117{\cdot}5$ feet for the length. Will this length be sufficient? As there are twenty-eight beds, or fourteen on each side, each bed would thus have $\frac{117{\cdot}5}{14} = 8{\cdot}4$ feet of wall-space, which would provide for a bed of 3 feet in width, and a space of $8{\cdot}4 - 3{\cdot}0 = 5{\cdot}4$ feet between each. As a minimum of only 3 feet between each bed has been fixed, 117 feet will amply satisfy the conditions as to length. But it may be remarked incidentally that although the beds might be put closer together, without overstepping the limit laid down, yet it would not be permissible on that account either to put more beds into the ward, or to shorten its length, otherwise each individual would be deprived of a portion of his floor-space and cubic space. The deficiency in the latter could be overcome by raising the height of the ward to 14 or 15 feet (not beyond this); but the deficiency in floor-space could not thus be overcome, except by increasing the width of the ward, which, of course, could be done. So the conditions are fulfilled in a ward 117·5 feet long, 24 feet wide, and 12 feet high, which provides a distance of more than 5 feet between each bed. If we choose to make the ward 25 feet wide, the length could then be $\frac{2,800}{25} = 112$ feet, which would provide a wall-space of $\frac{112}{14} = 8$ feet for each bed; this would be well over the minimum limit, and satisfy all conditions laid down.

Window-Space:

1 square foot to every 70 cubic feet of space. There must, therefore, be a minimum of $\frac{33,600}{70} = 480$ square feet of window-space. If the windows are placed on each side of the ward, 240 square feet of window-space on each side will be required. If a window be placed between each bed, fourteen beds on each side will require thirteen windows, each having a minimum area of $\frac{240}{13} = 18{\cdot}4$ square feet. If each window commences 3 feet from the floor, and reaches to within 1 foot of the ceiling, since height of ward is 12 feet, height of window must be 8 feet, and $\frac{18{\cdot}4}{8} = 2{\cdot}3$ feet will be the necessary width of window. These are the lowest limits allowable, and they might well be made somewhat wider. To what limit in width may we go? It has been said that there must be 4 feet of wall-space between the windows (or 1 foot more than the width of the bed). If width of bed is 3 feet, the required space between the windows would be left by bringing them to within 6 inches of the bed on each side; and since space between the beds $= 5{\cdot}4$ feet, the maximum width of window would be $5{\cdot}4$ feet $-$ (2×6 inches) $= 4{\cdot}4$ feet.

The **minimum** dimensions of the ward which will satisfy all conditions are, therefore:

Length, 117·5 feet; breadth, 24 feet; height, 12 feet; window-space, 480 square feet.

If it be decided to have a window between each bed (*i.e.*, twenty-six windows), the minimum size must be 8 feet high and 2·3 feet wide, and the maximum width 4·4 feet.

There are numerous other ways in which the ward could be constructed; moreover, the matter of ventilation has not been touched upon; but the object in view has been to point out, in as simple a manner as possible, the chief features to be borne in mind, and to suggest a method of 'setting to work.'

[NOTE.—For a *fever* ward, the allowance for each patient should be:

Floor-space = 140 square feet;
Cubic space = 2,000 cubic feet;
Wall-space = 12 linear feet.

Since the floor-space should be 140 square feet, the height of the ward should be at least $\frac{2,000}{140} = 14$ feet, to obtain the necessary 2,000 feet of cubic space.]

When estimating the air-space of a room, some deduction should be made for the space occupied by each person living in the room. The average number of cubic feet occupied by each person $= \frac{1}{4}$ (weight of person in stones).

Example:

How much window-space would be required for a room 15 feet long, 12 feet wide, and 12 feet high?*

$$\text{Cubic space of room} = 15 \times 12 \times 12 = 2,160 \text{ cubic feet.}$$

Allowing 1 square foot of window-space to every 70 cubic feet of room-space,

$$\therefore \frac{2,160}{70} = 31 \text{ square feet of window-space required.}$$

* D.P.H. Exam. (Roy. Coll. Phys. Surg.).

CHAPTER VIII

ALCOHOL AND PROOF-SPIRIT

Example:

What is proof-spirit? How much alcohol ought it to contain (i.) by volume, (ii.) by weight?*

The relationship of alcohol and proof-spirit by volume may be expressed as follows:

Alcohol	57 vols.
Water	43 "
Proof-spirit	100 "

That is, proof-spirit contains 57 per cent. of alcohol by volume.

Or,

$$A : P.S. :: 57 : 100;$$
$$:: 1 : 1{\cdot}76.$$

Therefore

$$P.S. = A \times 1{\cdot}76.$$

Similarly:

$$A = \frac{P.S.}{1{\cdot}76}.$$

So, to find the volume of proof-spirit, multiply the

* D.P.H. Exam. (Roy. Coll. Phys. Surg.).

volume of alcohol by 1·76; and, to find the volume of alcohol, divide the volume of proof-spirit by 1·76.

[Since 1·76 is approximately $1\frac{3}{4}$, or $\frac{7}{4}$, an almost identical result would be obtained by the simpler method of multiplying by $\frac{7}{4}$ or $\frac{4}{7}$ respectively.]

Overproof.—A spirit 'x° overproof' means that 100 vols. of the spirit will dilute with x vols. of water to form $100+x$ vols. of P.S. Therefore, to reduce it to proof, x vols. of water must be added to 100 vols. of the spirit.

Underproof.—'x° underproof' means that 100 vols. of the spirit contain only as much alcohol as $100-x$ vols. of P.S.

Examples:

(i.) A sample of whisky contains 70 per cent. by volume of alcohol. How much overproof is it?

$$\text{P.S.} = \text{A} \times 1{\cdot}76 = 70 \times 1{\cdot}76 = 122{\cdot}7.$$

That is, the sample is 22·7° overproof.

(ii.) A sample of whisky contains 43 per cent. by volume of alcohol. How much underproof is it?

$$\text{P.S.} = \text{A} \times 1{\cdot}76 = 43 \times 1{\cdot}76 = 75;$$

that is, it contains 75 per cent. of proof-spirit, and is, consequently, $100 - 75 = 25^\circ$ underproof.

(iii.) A sample is 30° overproof. What percentage of alcohol does it contain?

Being 30° overproof, it will dilute with water to form 130 vols. of P.S.

$$\text{A} = \frac{\text{P.S.}}{1{\cdot}76} = \frac{130}{1{\cdot}76} = 74.$$

It therefore contains 74 per cent. of alcohol by volume.

(iv.) How much alcohol by volume is contained in a sample of spirit 20° underproof?*

Being 20° underproof, it contains only as much alcohol as 80 vols. of P.S.

$$A = \frac{P.S.}{1{\cdot}76} = \frac{80}{1{\cdot}76} = 46.$$

It therefore contains 46 per cent. of alcohol by volume.

(v.) A sample of whisky is found to be 40 per cent. underproof. With how much water has it been fraudulently adulterated?

40° underproof means that it contains as much alcohol as 60 vols. of P.S. Now, under the Sale of Foods and Drugs Act Amendment Act, 1879, whisky may be sold 25° underproof—*i.e.*, containing only 75 per cent. proof-spirit.

It may therefore be said that—

75 per cent. P.S. = 100 per cent. 'legally pure' whisky.

$\therefore$ 60 per cent. P.S. = $\frac{100 \times 60}{75}$ = 80 per cent. 'pure' whisky.

The sample therefore contains 80 per cent. of whisky up to legal standard, and 20 per cent. added water.

[NOTE.—It follows from Example (ii.) that the Act of 1879 requires a sample of whisky to contain at least 43 per cent. of alcohol by volume.]

(vi.) How much spirit at 10° underproof must be added to a spirit at 30° overproof in order to produce a mixture of 5° overproof?

* D.P.H. Exam. (Roy. Coll. Phys. Surg.).

Let x = amount of spirit required at 10° underproof,

then x gallons contain $\frac{90x}{100}$ gallons P.S.

Let y = amount required at 30° overproof,

then y gallons are equivalent to $\frac{130y}{100}$ gallons P.S.

$$\therefore \frac{90x}{100} + \frac{130y}{100} = \text{amount of P.S. in mixture.}$$

But mixture required consists of $x+y$ gallons at 5° overproof,

and $x+y$ gallons $= \frac{105\,(x+y)}{100}$ gallons P.S.

$$\therefore \frac{90x}{100} + \frac{130y}{100} = \frac{105\,(x+y)}{100}$$

$$90x + 130y = 105x + 105y$$

$$25y = 15x;$$

$$\therefore x : y :: 25 : 15;$$

$$:: 5 : 3.$$

That is, 5 gallons of the weaker spirit must be added to 3 gallons of the stronger spirit to produce the required mixture.

This example may also be solved by the use of the 'rule' shown on p. 112.

Relationship between Volume and Weight.

Since specific gravity of alcohol = 0·793,

∴ 1 fluid oz. alcohol weighs 0·793 oz. (*say*, 0·8 oz.);

$$\therefore \text{vol.} : \text{weight} :: 1 : 0{\cdot}8;$$

$$\text{or, } \frac{\text{vol.}}{\text{weight}} = \frac{1}{0{\cdot}8} = 1{\cdot}25.$$

∴ percentage by vol. = percentage by weight × 1·25; and percentage by weight = percentage by volume × 0·8.

Therefore, to find percentage of alcohol by weight, multiply the percentage by volume by 0·8. And since proof-spirit contains 57 per cent. of alcohol by volume, it contains $57 \times 0{\cdot}8 = 45{\cdot}6$ per cent. by weight.

[NOTE.—This means 45·6 per cent. by weight in 100 volumes, not weight in 100 parts by weight. In 100 parts by weight of proof-spirit the alcohol weighs 49·24 parts.]

CHAPTER IX

CHEMICAL CALCULATIONS

Example:

Estimate the percentage of SO_2 in a room after disinection with sulphur, 2 lbs. of sulphur being burnt for every 1,000 cubic feet of air.

The equation $S_2 + 2O_2 = 2SO_2$ shows that 64 parts by weight of S combine with 64 parts of O, to produce 128 parts by weight of SO_2; that is, S produces twice its weight of SO_2, or 2 lbs. of S produce 4 lbs. of SO_2.

1 cubic foot dry air at 32° F. and 760 mm. = 566·85 grains (p. 3).

$\therefore$ 1 cubic foot SO_2 at 32° F. and 760 mm. $= \frac{566{\cdot}85 \times 32}{14{\cdot}47}$

= 1253·57 grains (p. 4).

$\therefore$ 1 cubic foot SO_2 at 60° F. (the ordinary temperature of a room) =

$$1253{\cdot}57 \times \frac{491}{491 + (60 - 32)} = 1185{\cdot}94 \text{ grains (p. 12)}$$

$$= \frac{1185{\cdot}94}{7{,}000} \text{ lbs.} = 0{\cdot}169 \text{ lb.};$$

that is, 1 cubic foot SO_2 = 0·169 lb.

$$\therefore 1 \text{ lb. } SO_2 = \frac{1}{0{\cdot}169} = 5{\cdot}9 \text{ cubic feet.}$$

$$\therefore 4 \text{ lbs. } SO_2 = 23{\cdot}6 \text{ cubic feet.}$$

But 4 lbs. SO_2 are produced by 2 lbs. of S.

$\therefore$ 2 lbs. S produce 23·6 cubic feet SO_2.

And as 2 lbs. S are burned for every 1,000 cubic feet of air, there are 23·6 cubic feet SO_2 in 1,000 cubic feet of air, or 2·36 per cent. SO_2.

Example:

What volume of sulphur dioxide (measured at 10° C. and 760 mm.) will be produced by the action of sulphuric acid on 30 grammes of copper?*

The chemical equation is as follows:

$$\underset{(63·5)}{Cu} + 2H_2SO_4 = CuSO_4 + \underset{(64)}{SO_2} + 2H_2O.$$

Since the density of $SO_2 = \frac{64}{2} = 32$ (p. 3),

$\therefore$ 32 grammes $SO_2 = 11·2$ litres (p. 3);

$\therefore$ 64 grammes $SO_2 = 22·4$ litres.

But the chemical equation shows that 64 grammes SO_2 are evolved by the action of H_2SO_4 on 63·5 grammes Cu.

$\therefore$ 63·5 grammes Cu produce 22·4 litres SO_2;

$\therefore$ 30 grammes Cu produce $\frac{22·4 \times 30}{63·5} = 10·58$ litres SO_2 at 0° C. and 760 mm.;

and 10·58 litres at 0° C. become $\frac{10·58 \times 283}{273} =$ 10·96 litres at 10° C. (p. 6).

Ans.: 10·96 litres.

Example:

How many grains of carbon are contained in 1,000 grains of glucose? †

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* D.P.H. Exam. (Roy. Coll. Phys. Surg.).

† *Ibid.*

Glucose = $C_6H_{12}O_6$.

$$C_6 = 12 \times 6 = 72$$
$$H_{12} = 1 \times 12 = 12$$
$$O_6 = 16 \times 6 = 96$$
$$180$$

Since there are 72 grains of carbon in 180 grains of glucose, how many will there be in 1,000?

$$180 : 1{,}000 : : 72 : x$$
$$\therefore x = \frac{1000 \times 72}{180} = 400.$$

That is, there are 400 grains of carbon in 1,000 grains of glucose.

STANDARD SOLUTIONS.

Silver Nitrate Solution for Testing for Chlorine.

$$AgNO_3 + NaCl = AgCl + NaNO_3.$$
$$(Ag = 108,\ Cl = 35{\cdot}5,\ AgNO_3 = 170).$$

This equation shows that 35·5 mgrs. Cl combine with 108 mgr. Ag; but 108 mgrs. Ag are contained in 170 mgrs. $AgNO_3$.

∴ 35·5 mgrs. Cl require 170 mgrs. $AgNO_3$ for chemical combination;

$$\therefore 1 \text{ mgr. Cl requires } \frac{170}{35{\cdot}5} = 4{\cdot}788 \text{ mgrs. } AgNO_3.$$

And since the solution is generally made of such a strength that 1 c.c. exactly neutralizes 1 mgr. Cl, each c.c. of solution must contain 4 788 mgrs. $AgNO_3$, or 1,000 c.c. (1 litre) must contain 4·788 grammes $AgNO_3$. The standard solution, therefore, is made by dissolving 4·788 grms. $AgNO_3$ in 1 litre of distilled water.

Ammonium Chloride Solution for 'Nesslerizing.'

The solution is generally made of such a strength that 1 c.c. = 0·01 mgr. of NH_3.

Since molecular weight of $NH_3 = 17$,
and " " $NH_4Cl = 53·5$,

therefore 17 mgrs. NH_3 are contained in 53·5 mgrs. NH_4Cl,

and 0·01 mgr. NH_3 is contained in $\frac{53·5 \times 0·01}{17} =$ 0·03147 mgr. NH_4 Cl.

Each c.c. therefore must contain 0·03147 mgr. NH_4Cl, or 0·03147 grammes per litre.

Solution for Estimation of CO_2 in Air.

Example :

In estimating the CO_2 in air by Pettenkofer's method, the oxalic acid solution is generally made of such a strength that 1 c.c. represents 0·5 c.c. CO_2. How is the solution made? The equation

$$H_2C_2O_4 + CaO = CaC_2O_4 + H_2O$$

shows that a molecule of oxalic acid neutralises a molecule of CaO; but molecular weight of oxalic acid ($H_2C_2O_4 + 2H_2O$) = 126, and that of CaO = 56,

∴ 126 parts of oxalic acid neutralise 56 parts of CaO.

Again, the equation

$$CO_2 + CaO = CaCO_3$$

shows that 44 parts of CO_2 neutralise 56 parts of CaO. Therefore with regard to their neutralising power on CaO

126 grms. oxalic acid = 44 grms. CO_2;
= 22·4 litres CO_2 (p. 3);

$$\therefore \frac{126}{22 \cdot 4} = 5 \cdot 625 \text{ grms. oxalic} = 1 \text{ litre } CO_2;$$

$\therefore$ 5·625 mgrms. oxalic = 1 c.c. CO_2;
$\therefore$ 2·813 ,, ,, = 0·5 c.c. CO_2.

And since each c.c. of solution is to represent 0·5 c.c. CO_2, each c.c. must contain 2·813 mgrms., or 2·813 grms. per litre.

Given Two Solutions of Different Strengths, to make a Third Solution of Intermediate Strength.

Let a = percentage strength of weaker solution;
b = ,, ,, stronger solution;
c = ,, ,, intermediate solution required.
Let x = the required amount of weaker solution;
y = ,, ,, stronger solution.
Then $x+y$ = the amount of intermediate solution formed by the mixture.

Since 100 parts of weaker solution contain a parts,

$$\therefore x \text{ parts contain } \frac{ax}{100} \text{ parts};$$

and since 100 parts of stronger solution contain b parts,

$$\therefore y \text{ parts contain } \frac{by}{100} \text{ parts};$$

$$\therefore x+y \text{ parts contain } \frac{ax}{100}+\frac{by}{100} \text{ parts.}$$

But 100 parts intermediate solution contain c parts,

$$\therefore x+y \text{ parts contain } \frac{c(x+y)}{100} \text{ parts.}$$

$$\text{Therefore, } \frac{c(x+y)}{100} = \frac{ax}{100} + \frac{by}{100};$$

$$\therefore\ cx + cy = ax + by;$$

$$x(c-a) = y(b-c),$$

$$\text{or } \frac{x}{y} = \frac{b-c}{c-a};$$

That is, the quantities x and y must be in the proportion of $(b-c) : (c-a)$ respectively, from which the following 'rule' is obtained:

The amount of weaker solution required is the difference between the percentage strengths of the stronger and intermediate solutions, whilst the amount of stronger solution required is the difference between those of the intermediate and weaker solutions.

Example:

To make a 1 : 40 carbolic acid solution from Acidum Carbolicum Liquefactum B.P.

Strength of ac. carb. liq. = 90 per cent.
" " 1 : 40 sol. = 2·5 per cent.
" " water = 0 per cent.

$\therefore$ amount of ac. carb. liq. = 2·5 − 0 = 2·5 parts;
" " water = 90 − 2·5 = 87·5 parts;

$$\therefore \frac{\text{water}}{\text{carb. acid}} = \frac{87{\cdot}5}{2{\cdot}5} = \frac{35}{1};$$

that is, 1 part of carbolic acid and 35 parts of water.

To Convert 'Parts per 100,000' into 'Grains per Gallon.'

x parts per 100,000 parts,

or $\frac{x \times 70{,}000}{100{,}000} = \frac{7x}{10}$ parts per 70,000 parts,

or $\frac{7x}{10}$ grains per 70,000 grains,

or $\frac{7x}{10}$ grains per gallon;

that is, x parts per 100,000 parts $= \frac{7x}{10}$ grains per gallon.

So, if 'grains per gallon' be multiplied by 10, and divided by 7, the result is 'parts per 100,000'; and if 'parts per 100,000' be multiplied by 7 and divided by 10, the result is 'grains per gallon.'

[NOTE.—1 grain per gallon = 1 part per 70,000 parts;
1 mgrm. per 100 c.c. = 1 part per 100,000 parts.]

Example:

Express 0·0042 and 0·0063 grain per gallon as parts per million.*

0·0042 grain per gallon

$= \frac{0{\cdot}0042 \times 10}{7} = 0{\cdot}006$ part per 100,000

$= 0{\cdot}06$ part per 1,000,000.

Also, 0·0063 grain per gallon

$= \frac{0{\cdot}0063 \times 10}{7} = 0{\cdot}009$ part per 100,000

$= 0{\cdot}09$ part per 1,000,000.

* D.P.H. Exam. (Roy. Coll. Phys. Surg.).

To Convert 'Grains per Gallon' into 'Parts per 100,000.'

1 gallon of water = 70,000 grains.

If there are x grains per gallon, there are

x grains in 70,000 grains,

or $\frac{x \times 100{,}000}{70{,}000} = \frac{10x}{7}$ grains in 100,000 grains,

or $\frac{10x}{7}$ parts per 100,000 parts;

that is, x grains per gallon $= \frac{10x}{7}$ parts per 100,000 parts.

NORMAL SOLUTIONS.

A *normal solution* is one which contains the hydrogen equivalent of the active substance in grammes per litre.

From the equation

$$HCl + NaOH = NaCl + H_2O$$

it is seen that 36·5 parts of HCl neutralize 40 parts of NaOH; so that if 36·5 grammes of HCl be added to a litre of water, and 40 grammes of NaOH to another litre, then equal volumes of these two solutions, when mixed together, will neutralize each other; that is, the solutions are chemically equivalent. These are termed 'normal' ($\frac{N}{1}$) solutions. From this it follows that a given volume of *any* normal acid solution will exactly neutralize the same volume of *any* normal alkaline solution.

Again, the equation

$$2HCl + Na_2CO_3 = 2NaCl + CO_2 + H_2O$$

shows that 73 parts of HCl neutralize 106 parts of Na_2CO_3, or 36·5 parts HCl neutralize 53 parts Na_2CO_3; that is, 1 molecule of monobasic HCl can neutralize only

$\frac{1}{2}$ a molecule of bivalent Na_2CO_3; hence $\frac{N}{1}$ solution of Na_2CO_3 contains $\frac{106}{2}=53$ grammes per litre.

Similarly, $\frac{N}{1}$ solution of acetic acid ($H.C_2H_3O_2$), which is monobasic (*i.e.*, contains only 1 molecule of displaceable hydrogen), contains $1+24+3+32=60$ grammes per litre.

$\frac{N}{1}$ solution of tartaric acid ($H_2.C_4H_4O_6$), which is dibasic, contains $\frac{2+48+4+96}{2}=\frac{150}{2}=75$ grammes per litre.

And $\frac{N}{1}$ solution of citric acid ($H_3.C_6H_5O_7$), which is tribasic, contains $\frac{3+72+5+112}{3}=\frac{192}{3}=64$ grammes per litre.

Example:

The acidity of 10 c.c. of a sample of beer is found to be neutralized by 3 c.c. of decinormal ($\frac{N}{10}$) solution of NaOH. Express the acidity in terms of glacial acetic acid.

From what has been said as to the nature of normal solutions, it is obvious that 3 c.c. of $\frac{N}{10}$ NaOH neutralize 3 c.c. of $\frac{N}{10}$ acetic; therefore the acidity of the sample is equivalent to 3 c.c. of $\frac{N}{10}$ acetic.

But—

$\frac{N}{1}$ acetic contains 60 grammes per litre,
or 60 mgrs. per c.c.

$\therefore$ $\frac{N}{10}$ acetic contains 6 mgrs. per c.c.
$\therefore$ 3 c.c. contain $3\times 6=18$ mgrs. acetic acid.

Therefore the acidity of 10 c.c. of the beer = 18 mgrs. acetic acid, or 180 mgrs. per 100 c.c.

The acidity is sometimes stated as grains per pint, thus:

180 mgrs. per 100 c.c.

$= 180$ parts per 100,000 parts

$= \frac{180 \times 7}{10} = 126$ grains per gallon (p. 113).

$= \frac{126}{8} = 15{\cdot}75$ grains per pint.

Example:

A loaf was found to consist of 33 per cent. of crust, and of 67 per cent. of crumb; 1 c.c. of $\frac{N}{10}$ alkaline solution was required to neutralize the acidity of 10 grammes of crust, and $1{\cdot}75$ c.c. $\frac{N}{10}$ alkaline solution were required to neutralize the acidity of 10 grammes of crumb. Calculate the acidity of the whole loaf, and express the result in terms of acetic acid.*

The data given provide only for the percentage, not the actual acidity being estimated. It must be assumed, therefore, that the loaf weighs 100 grammes, and consists of 33 grms. of crust, and 67 grms. of crumb.

10 grms. crust are neutralized by 1 c.c. $\frac{N}{10}$ alkaline sol.; but 1 c.c. $\frac{N}{10}$ acetic is also neutralized by 1 c.c. $\frac{N}{10}$ alk. sol.,

∴ acidity of 10 grammes crust $= 1$ c.c. $\frac{N}{10}$ acetic;

∴ " 33 " " $= \frac{33}{10} = 3{\cdot}3$ c.c. acetic.

10 grms. crumb are neutralized by $1{\cdot}75$ c.c. $\frac{N}{10}$ alk. sol.,

∴ acidity of 10 grms. crumb $= 1{\cdot}75$ c.c. $\frac{N}{10}$ acetic;

∴ " 67 grms. " $= \frac{1{\cdot}75 \times 67}{10} = 11{\cdot}7$ c.c. acetic.

Therefore, acidity of $(33 + 67 =)$ 100 grammes loaf $=$
$3{\cdot}3 + 11{\cdot}7 = 15$ c.c. $\frac{N}{10}$ acetic.

But each c.c. of $\frac{N}{10}$ acetic contains 6 mgrs.; (p. 115).

∴ 15 c.c. contain $15 \times 6 = 90$ mgrs. acetic acid;

∴ acidity of 100 grammes loaf $= 90$ mgrs. acetic acid

$= 0{\cdot}09$ grms. " "

* D.P.H. Exam. (Roy. Coll. Phys. Surg.).

Acidity of loaf, therefore = 0·09 per cent.

The acidity of bread is sometimes stated in grains per lb., thus :—

0·09 per cent. = 0·09 grs. per 100 grs.
= 0·09 × 70 grs. per 7000 grs.
= 6·3 grs. per 7000 grs.
= 6·3 grs. per lb.

CHAPTER X

LOGARITHMS

It is impossible in a book of this description, nor is it necessary, to enter into a detailed explanation of logarithms, or to describe their various uses; but if the reader will master the few points here given, he should have no difficulty in making use of the logarithmic (or Registrar-General's) method of estimating a population.

Definition.—A logarithm of a number to a given base is the index of the power to which the base must be raised to make it equal to the number.

For example, the log of 100 to the base 10 (written log_{10} 100) is 2, since 2 is the index of the power to which the base 10 must be raised to make it equal to the number 100. That is—

$$log_{10}\ 100 = 2, \text{ since } (10)^2 = 100.$$

Similarly:

$$log_{10}\ 1000 = 3, \text{ since } (10)^3 = 1000,$$
$$\text{and } log_{10}\ 10 = 1, \text{ since } (10)^1 = 10,$$
$$\text{and } log_{10}\ 1 = 0, \text{ since } (10)^0 = 1.$$

In common logs the base is always 10, and may, therefore, be omitted. Thus, '*log* 100' is understood to mean 'log_{10} 100.'

To obtain a working knowledge of logarithms, it is only essential to remember the four fundamental properties which they possess (without burdening the mind as to the proof thereof). These are—

$$\text{(i.)}\quad \log(a \times b) = \log a + \log b.$$

$$\text{(ii.)}\quad \log\left(\frac{a}{b}\right) = \log a - \log b.$$

$$\text{(iii.)}\quad \log(a)^n = n \log a.$$

$$\text{(iv.)}\quad \log \sqrt[n]{a} = \frac{\log a}{n}.$$

The four following examples will illustrate the use of these properties :

(i.) **Multiplication of numbers can be performed by addition of their logarithms.**

Example :

Multiply 203 by 45.

$$\text{Since } \log(a \times b) = \log a + \log b,$$
$$\therefore \log(203 \times 45) = \log 203 + \log 45.$$

From a table it is found that—

$$\log 203 = 2{\cdot}3074960$$
$$\log 45 = 1{\cdot}6532125$$
$$\text{By addition} \quad 3{\cdot}9607085$$
$$\therefore \log(203 \times 45) = 3{\cdot}9607085.$$

But from the table—

$$3{\cdot}9607085 = \log 9135\,;$$
$$\therefore \log(203 \times 45) = \log 9135\,;$$
$$\therefore 203 \times 45 = 9135.$$

(The method of obtaining these logs from the table is explained below.)

(ii.) **Division of numbers can be performed by subtraction of their logs.**

Example :

Divide 203 by 45.

$$\text{Since } log\left(\frac{a}{b}\right)=log\ a-log\ b,$$

$$\therefore\ log\left(\frac{203}{45}\right)=log\ 203-log\ 45.$$

From the table—

$$log\ 203=2{\cdot}3074960$$
$$log\ \ 45=1{\cdot}6532125$$

$$\text{By subtraction}\quad 0{\cdot}6542835$$

$$\therefore\ log\left(\frac{203}{45}\right)=0{\cdot}6542835.$$

But from the table it is found that—

$$0{\cdot}6542835=log\ 4{\cdot}51\ ;$$

$$\therefore\ log\left(\frac{203}{45}\right)=log\ 4{\cdot}51\ ;$$

$$\therefore\ \frac{203}{45}=4{\cdot}51.$$

(iii.) **Involution of numbers, or the process by which the powers of quantities are obtained, can be performed by multiplication of their logs.**

Example :

Find the fourth power of 13; that is, find the value of $(13)^4$.

$$\text{Since } log\ (a)^n=n\ log\ a,$$
$$\therefore\ log\ (13)^4=4\ log\ 13.$$

From a table, it is found that—

$$log\ 13=1{\cdot}1139434$$
$$\text{Multiply by 4}\qquad 4$$

$$\therefore\ 4\ log\ 13=4{\cdot}4557736$$
$$\therefore\ log\ (13)^4=4{\cdot}4557736.$$

But by table—

$$4{\cdot}4557736 = \log 28561;$$
$$\therefore \log (13)^4 = \log 28561;$$
$$\therefore (13)^4 = 28561.$$

(iv.) **Evolution of numbers, or the extraction of roots, can be performed by division of their logs.**

Example:

Find the cube root of 3,375.

$$\text{Since } \log \sqrt[n]{a} = \frac{\log a}{n},$$
$$\therefore \log \sqrt[3]{3375} = \frac{\log 3375}{3}.$$

From table—

$$\log 3375 = 3{\cdot}5282738;$$
$$\therefore \frac{\log 3375}{3} = 1{\cdot}1760913;$$
$$\therefore \log \sqrt[3]{3375} = 1{\cdot}1760913.$$

But from table—

$$1{\cdot}1760913 = \log 15;$$
$$\therefore \log \sqrt[3]{3375} = \log 15;$$
$$\therefore \sqrt[3]{3375} = 15.$$

METHOD OF USING THE TABLES.

A logarithm consists of two parts, an integral part before the decimal point, and a decimal part after the point.

Thus, *log* 1019 = 3·0081742, of which the figure 3 is the integral part (called the 'characteristic' or 'index'), and 0081742 the decimal part (called the 'mantissa').

In the tables **only the decimal part is given**; the index must be prefixed according to rule, as explained below.

Supposing the logs of the following numbers are required: 1·019, 10·19, 101·9, 1019, and 10190. It will be noticed that the digits of these five numbers are all similar, the only difference being the position of the decimal point. **Where the digits of the numbers are the same, the mantissa of the logarithm is the same,** no matter where the decimal point is placed. It is different with the characteristic, which is always **one less than the number of integral figures in the number.** Thus, the above five numbers have respectively 1, 2, 3, 4, and 5 integral figures (*i.e.*, figures in front of the decimal point); the characteristics of their logs, therefore, are 0, 1, 2, 3, and 4 respectively, whilst the mantissa is the same in all of them.

Therefore—

log 1·019=0·0081742
log 10·19=1·0081742
log 101·9=2·0081742
log 1019=3·0081742
log 10190=4·0081742

Thus, the mantissa is altogether independent of the position of the decimal point, whilst the characteristic is decided solely by that position.

No.		0	1	2	3	4	5	6	7	8	9
1014	006	0380	0808	1236	1664	2092	2521	2949	3377	3805	4233
15		4660	5088	5516	5944	6372	6799	7227	7655	8082	8510
16		8937	9365	9792	$\overline{0219}$	$\overline{0647}$	$\overline{1074}$	$\overline{1501}$	$\overline{1928}$	$\overline{2355}$	$\overline{2782}$
17	007	3210	3637	4064	4490	4917	5344	5771	6198	6624	7051
18		7478	7904	8331	8757	9184	9610	$\overline{0037}$	$\overline{0463}$	$\overline{0889}$	$\overline{1316}$
19	008	1742	2168	2594	3020	3446	3872	4298	4724	5150	5576

The above is an extract from a table of logarithms embracing all numbers up to five figures, the first four of which will be found in the left-hand column, and the fifth at the top. The first line of the table, opposite the number 1014, reads as follows: 0060380, 0060808, 0061236, etc., the three figures (006) at the commencement of the line applying throughout. In the line opposite the number 1016 it will be noticed that some of the figures have a 'vinculum,' or line over them, which means that all such figures belong to the next (viz., the 007) group. This line, then, reads as follows: 0068937, ... 0069792, 0070219, 0070647, etc. Similarly, the line opposite the number 1018 reads: 0077478 . . . . . . 0079610, 0080037, etc.

To find the Log of a Given Number from the Table.

Example:

From the table given above, find the log of 101·76.

First find the log of 10176, neglecting the decimal point. Opposite 1017 in the first column, and under the figure 6 at the top, will be found 5771; prefix the three figures 007, and 0075771 is the mantissa required. What characteristic should there be? The number 101·76 has three integral figures, therefore the characteristic will be 2; hence—

$$\log 101{\cdot}76 = 2{\cdot}0075771$$

(whilst *log* 1017·6 would be 3·0075771).

Example:

Find the log of 1·0187.

Ans.: 0·0080463.

To find the Number corresponding to a Given Log.

Example:

From the given table find the number whose log is 1·0069792.

Look in the '006 group' for the figures 9792, which will be found opposite the number 1016, and under the figure 2 at the top. The figures required, therefore, are 10162. Where must the decimal point be placed? Since the characteristic of the given log is 1, the number of integral figures must be two; place the decimal point accordingly, and 10·162 is the number required; that is—

$$\log 10{\cdot}162 = 1{\cdot}0069792.$$

Example:

Find the number whose log is 2·0072355.

Ans.: 101·68.

A table of logs usually extends to numbers of five figures only. Where the numbers contain more than five figures, the procedure is as follows:

To find the Log of a Given Number which is not Contained in the Tables.

Example:

Required the log of 1015543.

This number lies between 1015500 and 1015600. Take from the table the logs of the first five figures; that is, find the logs of 10155 and 10156, thus:

$$\log 10156 = 4{\cdot}0067227$$
$$\log 10155 = 4{\cdot}0066799$$

Therefore, $\log 1015600 = 6{\cdot}0067227$
and $\log 1015500 = 6{\cdot}0066799$

By subtraction $100 = 0{\cdot}0000428$

Thus, an increase of 100 in the number produces an increase of 0·0000428 in the log. But the difference between 1015543 (the number whose log is required) and 1015500 is only 43. What increase will there be in the log corresponding to an increase of 43 in the number?

$$100 : 43 :: 0{\cdot}0000428 : x$$

$$x = \frac{0{\cdot}0000428 \times 43}{100} = 0{\cdot}0000184.$$

$$\begin{aligned} \text{So, } \log 1015500 &= 6{\cdot}0066799 \\ \text{Add } \quad 43 &= 0{\cdot}0000184 \\ \hline \text{Therefore, } \log 1015543 &= 6{\cdot}0066983 \end{aligned}$$

∴ 6·0066983 is the log required.

Example:

Find the log of 101·4736.

Ans.: 2·0063531.

To find the Number corresponding to a Given Log which is not exactly contained in the Table.

Example:

Find the number whose log is 3·0070421.

On reference to the table, the mantissa of this log will be seen to lie between 0070219 and 0070647, and the numbers opposite these logs are 10163 and 10164 respectively. Since the characteristic of the given log is 3, the number required will have four integral figures. Place the decimal point accordingly, thus:

$$\begin{aligned} \log 1016{\cdot}4 &= 3{\cdot}0070647 \\ \log 1016{\cdot}3 &= 3{\cdot}0070219 \\ \hline \text{Difference, } \quad 0{\cdot}1 &= 0{\cdot}0000428 \end{aligned}$$

Thus, an increase of 0·0000428 in the log produces an increase of 0·1 in the number. But the difference between 3·0070421 (the log whose number is required) and 3·0070219 is only 0·0000202. What increase in the number will an increase of 0·0000202 produce?

$$0{\cdot}0000428 : 0{\cdot}0000202 :: 0{\cdot}1 : x$$

$$x = \frac{0{\cdot}0000202 \times 0{\cdot}1}{0{\cdot}0000428} = 0{\cdot}047.$$

So, $\quad \log 1016{\cdot}3 = 3{\cdot}0070219$

add $\quad {\cdot}047 = 0{\cdot}0000202$

$\therefore \log 1016{\cdot}347 = 3{\cdot}0070421$

$\therefore$ 1016·347 is the number required.

Example:

Find the number whose log is 1·0078925.

Ans.: 10·18339.

In practice this labour is avoided by the use of the table of 'proportional parts,' in which the necessary multiplication or division is already effected.

To find the Log of a Number consisting wholly of a Decimal.

Find $\log \frac{5}{8}$.

$$\log \tfrac{5}{8} = \log 5 - \log 8;$$
$$= 0{\cdot}69897 - 0{\cdot}90309;$$
$$= -0{\cdot}20412.$$

But $\frac{5}{8} = 0{\cdot}625$, and if the log of 625 be sought in a table, the mantissa given there will be found to be 79588.

The explanation of this is that in logarithms *the mantissa is always positive*, whilst the index may be either positive or negative.

Therefore $-0{\cdot}20412$ must be expressed in such a way that its mantissa shall be positive.

Thus, $-0{\cdot}20412$ may be written

$$\begin{aligned} &(-1+1)-0{\cdot}20412 \\ =& -1+(1-0{\cdot}20412) \\ =& -1+0{\cdot}79588 \\ =& \bar{1}{\cdot}79588 \end{aligned}$$

(the negative sign being placed *above* the index, to indicate that it refers to that figure alone).

$$\begin{aligned} \text{So, } \log 625 &= 2{\cdot}79588\,; \\ \log 62{\cdot}5 &= 1{\cdot}79588\,; \\ \log 6{\cdot}25 &= 0{\cdot}79588\,; \\ \log 0{\cdot}625 &= \bar{1}{\cdot}79588\,; \\ \log 0{\cdot}0625 &= \bar{2}{\cdot}79588\,; \end{aligned}$$

It is thus seen that the mantissa is always positive, whether the index is positive or negative.

It has been already shown (p. 122) how to find the index in the case of numbers containing one or more integral figures. Where the number consists wholly of a decimal portion, the rule is: Count from the decimal point to the first significant figure; this gives the index required, which is negative; thus, *log* 0·625 has an index of $\bar{1}$, and the index of *log* 0·00625 is $\bar{3}$, because the first significant figure (viz., 6) is the third from the decimal point.

To find the Number corresponding to a Given Log with a Negative Index.

Example:

Find the number whose log is $\bar{3}{\cdot}0080889$.

If the mantissa of this log be looked for in the table on p. 122, the number will be found to be 10188; place the decimal point and sufficient cyphers following it to make the first figure (viz., 1) the third from the decimal point, thus: 0·0010188;

that is, $\log 0{\cdot}0010188 = \bar{3}{\cdot}0080889$.

CHAPTER XI

VITAL STATISTICS

ESTIMATION OF POPULATION.

THE increase in a population takes place in Geometrical Progression.

[NOTE.—(*a*) Quantities are said to be in *arithmetical progression* when they increase or decrease by a common difference; thus, the numbers 2, 4, 6, 8, 10 are in A.P., the common difference being 2.

(*b*) Quantities are said to be in *geometrical progression* when each is equal to the product of the preceding and some constant factor; thus, the numbers 2, 4, 8, 16, 32 are in G.P., the constant factor (or common ratio) being 2.]

Assume that in the year 1901 the population of a town was 10,000, and 13,000 in 1911. There is an increase of 3,000 in the ten years, or an *average* of 300 annually. But it would be incorrect to say that 10,000 persons become 10,300 by the end of the first year, 10,600 by the end of the second year, and so on, to 13,000 in the tenth year; for the population is increasing year by year, owing to increased number of parents each year, and increased number attaining a marriageable age, and yet this increasing number only produces the same stationary annual increase of 300. For instance, if in the first year 10,000 produce an additional 300, making a total of 10,300, the 10,300 commencing the second year (if increasing in the same ratio, which is presumed to be the case) ought to produce more than 300—*viz.* $\frac{300 \times 10{,}300}{10{,}000} = 309$, and similarly for any other year. In the accompanying table, the population in Example 1 is

Year.	*Example 1 (in A.P.).*		*Example 2 (in G.P.).*	
	Population.	*Annual Increase.*	*Population.*	*Annual Increase.*
1901	10,000		10,000	
		300		266
1902	10,300		10,266	
		300		273
1903	10,600		10,539	
		300		280
1904	10,900		10,819	
		300		287
1905	11,200		11,106	
		300		296
1906	11,500		11,402	
		300		303
1907	11,800		11,705	
		300		311
1908	12,100		12,016	
		300		319
1909	12,400		12,335	
		300		329
1910	12,700		12,664	
		300		336
1911	13,000		13,000	

obtained by inserting 9 arithmetical means, and that in Example 2 by inserting 9 geometrical means between 10,000 and 13,000. The column headed 'Annual Increase' shows the difference between the population of any year and that of the preceding one. It will be noticed in Example 2 that the annual addition to the population increases with each succeeding year, as would naturally be the case; *e.g.*, if 10,000 produce 266, the 10,266 commencing the second year would produce $\frac{266 \times 10266}{10000} = 273$.

This example, therefore, gives the correct estimation of the population, whilst Example 1 shows an incorrect one. It is thus seen that the increase in population takes place in Geometrical Progression.

[The reader will doubtless observe the analogy between these two examples and similar instances of simple and compound interest respectively.

If £10,000 be invested at 3 per cent. simple interest, the income will be £300 per annum, or £3,000 for the 10 years; whereas if it be put out at 2·66 per cent., the first year's interest will only be £266. But if the interest be capitalised each year—that is, if the original sum be invested at 2·66 per cent. compound interest—the amount at the end of 10 years will be £13,000.]

Methods of Estimating a Population.

There are three methods made use of for the purpose of estimating a population.

Method 1:

This method estimates the population from the average birth-rate of the last ten years.

Example:

The average birth-rate of a town for the ten years (1907-1916) was 29·2 per 1,000. The actual number of births registered in 1917 was 440. Find the population for 1917.

There were 29·2 births in 1000 persons,

$$\text{or 1 birth in } \frac{1000}{29{\cdot}2},$$

$$\text{and 440 births in } \frac{440 \times 1000}{29{\cdot}2} = 15{,}066;$$

$\therefore$ estimated population for 1917 = 15,066.

$$\text{That is, population} = \frac{\text{registered births for the year} \times 1{,}000}{\text{average birth-rate for last 10 years}}.$$

Method 2:

This method estimates the population from the number of inhabited houses.

Example:

At the census of 1911 the population of a town was 11,316, and the number of inhabited houses was 1878, giving $\frac{11316}{1878} = 6{\cdot}012$ persons to each house. In 1917 the number of inhabited houses (as ascertained from the rate-book) was 2,014. If this number be multiplied by the average number of persons to each house, the product will give the estimated population for 1917—viz., $(2{,}014 \times 6{\cdot}012 =)$ 12,108.

$\therefore$ estimated population for 1917 = 12,108.

Method 3:

This method (known as the Registrar-General's) assumes that the rate of increase remains constant, and is the same as existed during the previous intercensal period of ten years.

Example:

Explain the process by which the population of a district is estimated from the last two census returns.*

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* D.P.H. Exam. (Roy. Coll. Phys. Surg.).

Let P = population in any census year, and
r = annual increase per unit of population.

Then, by the end of the year, one person becomes $1+r$, and P persons will become $P(1+r)$. The second year starts with this increased population—viz., $P(1+r)$, and thus

if 1 person becomes $1+r$, then
$P(1+r)$ persons become $P(1+r)\times(1+r)=P(1+r)^2$.
Similarly, at the end of the ninth year there will be $P(1+r)^9$ persons, and at the end of the tenth year they will have become $P(1+r)^9\times(1+r)=P(1+r)^{10}$, or after n years $P(1+r)^n$.

So, if P′ = population required, then

$$P'=P(1+r)^n.$$

The rate of increase (r) during any intercensal period of ten years can thus be found from this formula, by putting $n=10$, P′ = the population at the later census, and P = the population at the earlier census, To convert this into a form suitable for solution, take *logs* of each side, thus:

$$\begin{aligned} \log P' &= \log[P\times(1+r)^n] \\ &= \log P + \log(1+r)^n \\ &= \log P + n\log(1+r) \\ \therefore\ n\log(1+r) &= \log P' - \log P \\ \text{and } \log(1+r) &= \frac{\log P' - \log P}{n} \\ &= \frac{\log P' - \log P}{10}. \end{aligned}$$

Next, it is assumed that this same rate of increase continues to hold good up to the next census, so that if P″ =

the required population for any year, then, as before, $P'' = P'(1+r)^n$, and $log P'' = log P' + n\ log(1+r)$;

but it has been shown that $log(1+r) = \frac{log P' - log P}{10}$;

$$\therefore \log P'' = \log P' + n.\frac{\log P' - \log P}{10}.$$

This formula furnishes the following practical rule for estimating a population when the figures for the last two censuses are given:

(i.) From *log* of last census (P′) subtract *log* of previous census (P).

(Difference = *log* of the *ten* years' increase.)

(ii.) Divide by 10.

(Quotient = *log* of *annual* increase.)

(iii.) Multiply by the number of years (n) between the last census and the middle of the year sought.

(Product = *log* of the increase for those years.)

(iv.) Add *log* of last census (P′).

(Sum = *log* of the population required (P″).

The census is taken at the beginning of April, but all populations should be estimated to the middle of the year, or three months later; that is, the 'central' populations must always be found.

Example:

If P represents the population of a town at the census of 1871, and P′ its population at that of 1881, what will represent its calculated population for Midsummer, 1890?*

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* D.P.H. Exam. (Roy. Coll. Phys. Surg.).

Here P″ = population required for middle of 1890, and $n = 9\frac{1}{4}$ years (*i.e.*, from April 1881 to Midsummer 1890);

$$\therefore \textit{log}\text{ population required} = \textit{log}\,P' + 9\tfrac{1}{4}.\frac{\textit{log}\,P' - \textit{log}\,P}{10}.$$

Example:

The population of a town at the census of 1901 was 9,935, and at that of 1911 was 12,915; find the population for 1917 (*i.e.*, $6\frac{1}{4}$ years later).

log pop. 1911 = *log* 12,915	= 4·1110948
log pop. 1901 = *log* 9,935	= 3·9971514
difference = *log* of 10 years' increase	= 0·1139434
divide by 10 = *log* annual increase	= 0·0113943
multiply by $6\frac{1}{4}$	= 6·25
product = *log* of $6\frac{1}{4}$ years' increase	= 0·0712140
*add *log* pop. 1911 = *log* 12,915	= 4·1110948
∴ *log* pop. 1917	= 4·1823088

But 4·1823088 = *log* 15,216,
∴ pop. for 1917 = 15,216.

By giving to n (in the formula on p. 133) successive values of $\frac{1}{4}$, $1\frac{1}{4}$, $2\frac{1}{4}$, etc., the central populations for each year are obtained; or, what is the same thing, the population for any year can be obtained by adding $\frac{\textit{log}\,P' - \textit{log}\,P}{10}$ (the *log* of annual increase) to the *log* of the population of the preceding year.

* If the required population had been that for 1907—*i.e.*, $6\frac{1}{4}$ years after the *former* census—then 0·0712140 (the *log* of $6\frac{1}{4}$ years' increase) would have to be added to *log* population 1901, instead of, as here, to *log* population 1911.

Example:

Using the census figures of the preceding example, find the populations for each of the years of the intercensal period 1901–1910.

In this example $\frac{log P' - log P}{10}$, or *log* of annual increase, has already been found to be 0·0113943, and if this be added to the *log* of any one year's population, the sum will give the *log* of the next year's population. For the year 1891 only $\frac{1}{4}$ of the *log* of annual increase must be added, corresponding to the three months between the census and the middle of that year. Thus:

log census pop. 1901	=3·9971514	(as seen above)
add ($\frac{1}{4}$ × 0·0113943)	=0·0028486	
∴ *log central* pop. 1901	=4·0000000	(=*log* 10,000)
add	0·0113943	
∴ *log* pop. 1902	=4·0113943	(=*log* 10,266)
add	0·0113943	
∴ *log* pop. 1903	=4·0227886	(=*log* 10,539)
add	0·0113943	
∴ *log* pop. 1904	=4·0341829	(=*log* 10,819)
	etc.	

Therefore the central populations for the years 1901-1910 are: 10,000, 10,266, 10,539, 10,819, etc., and will be found tabulated in full on p. 129 (Example 2).

Example:

Population at 1901 census = 15,126;
" 1911 " = 18,472.
Find the population for 1919.
Ans.: 21,783.

To find the Mean Annual Population for a Period of Ten Years.

Estimate to the middle of the year the populations for each of the ten years, and take their sum ; this gives the total population for the ten years. Divide by 10, and the quotient will be the mean annual population. Using the previous example, and adding up the central populations for the ten years 1901-1910 inclusive, as set out on p. 129 (Example 2), their sum will be found to be 112,852 ; the mean annual population will thus be $\frac{112852}{10} = 11285{\cdot}2$.

The sum of the populations for ten years may, however, be found without actually estimating the populations for the individual years, as follows :

Let P = central population for 1901 ; then the sum of the successive populations for the ten years 1901-1910 will be (p. 132) :

$$\underset{(1901)}{P} + \underset{(1902)}{P(1+r)} + \underset{(1903)}{P(1+r)^2} + \underset{(1904)}{P(1+r)^3} + \ldots\ldots + \underset{(1910)}{P(1+r)^9}.$$

This is a geometrical progression consisting of 10 terms, P being the first term, and $(1+r)$ the common ratio. If the sum of these terms be found by the usual algebraical method, the total will be $\frac{P(1+r)^{10} - P}{(1+r) - 1}$.

But $P(1+r)^{10}$ = central pop. for 1911, and P = central pop. for 1901 :

$$\therefore \left\{\begin{matrix}\text{sum of 10} \\ \text{years' pop.} \\ \text{(1901-1910)}\end{matrix}\right\} = \frac{\left\{\begin{matrix}\text{central pop.} \\ \text{for 1911}\end{matrix}\right\} - \left\{\begin{matrix}\text{central pop.} \\ \text{for 1901}\end{matrix}\right\}}{r}.$$

Example :

According to the census returns, the population of a place for 1901 was 4,647, and for 1911 was 5,811. Find the mean annual population for the ten years 1901-1910.

(i.) First find the central populations for 1901 and 1911.

The usual formula (p. 133) applies, viz. :

$$\left\{\begin{matrix}\textit{log}\text{ central}\\ \text{pop. 1901}\end{matrix}\right\} = \left\{\begin{matrix}\textit{log}\\ \text{pop.}\\ 1901\end{matrix}\right\} + \frac{1}{4}\cdot\frac{\textit{log}\text{ pop. 1911} - \textit{log}\text{ pop. 1901}}{10},$$

$$\textit{i.e.,}\ \left\{\begin{matrix}\textit{log}\text{ central}\\ \text{pop. 1901}\end{matrix}\right\} = \textit{log}\ 4647 + \frac{1}{4}\cdot\frac{\textit{log}\ 5811 - \textit{log}\ 4647}{10}$$

This works out as follows :

log 5811	=3·7642509
log 4647	=3·6671727
difference	=0·0970782 (=*log* 10 years' increase)
divide by 10	=0·0097078 (=*log* annual increase)
multiply by $\frac{1}{4}$	=0·0024269 (=*log* $\frac{1}{4}$ year's increase)
add *log* 4647	=3·6671727
∴ *log* central pop. 1901	=3·6695996 (=*log* 4673)

∴ central pop. for 1901 = 4,673.

Similarly,

$$\left\{\begin{matrix}\textit{log}\text{ central}\\ \text{pop. 1911}\end{matrix}\right\} = \left\{\begin{matrix}\textit{log}\\ \text{pop.}\\ 1911\end{matrix}\right\} + \frac{1}{4}\cdot\frac{\textit{log}\text{ pop. 1911} - \textit{log}\text{ pop. 1901}}{10}.$$

log pop. 1911 = *log* 5811 = 3·7642509

add *log* $\frac{1}{4}$ year's increase = 0·0024269 (as above)

∴ *log* central pop. 1911 = 3·7666778 (= *log* 5844)

∴ central pop. for 1911 = 5,844.

(ii.) Find the value of r.

$$\log(1+r)=\frac{\log 5811-\log 4647}{10} \text{ (p. 132)};$$

but $$\frac{\log 5811-\log 4647}{10}=\log \text{ annual increase}$$

$$=0{\cdot}0097078 \text{ (\textit{vide} above)};$$

$$\therefore \log(1+r)=0{\cdot}0097078.$$

And from a table it is found that

$$0{\cdot}0097078=\log 1{\cdot}0226;$$

$$\therefore 1+r=1{\cdot}0226;$$

$$\therefore r=0{\cdot}0226.$$

And since

$$\left\{\begin{matrix}\text{total pop.}\\ \text{for 10}\\ \text{years}\end{matrix}\right\}=\frac{\left\{\begin{matrix}\text{central pop.}\\ \text{for 1911}\end{matrix}\right\}-\left\{\begin{matrix}\text{central pop.}\\ \text{for 1901}\end{matrix}\right\}}{r}$$

$$\therefore \left\{\begin{matrix}\text{total pop. for}\\ \text{the 10 years}\end{matrix}\right\}=\frac{5844-4673}{0{\cdot}0226}=51{,}814;$$

and mean annual pop. for the 10 years 1901-1910.

$$=\frac{51814}{10}=5181{\cdot}4.$$

If the same Rate of Increase continues, how long will a Population take to Double Itself?

$P'=P(1+r)^n$, and the population will be doubled when $P'=2P$;

$$\text{that is, } P(1+r)^n=2P, \therefore (1+r)^n=2;$$

$$\therefore n \log(1+r)=\log 2;$$

$$\therefore n=\frac{\log 2}{\log(1+r)}.$$

Applying this to the example on p. 134,

$$\log(1+r) = \log \text{ annual increase} = 0{\cdot}0113943\,;$$

$$\log 2 = 0{\cdot}3010300\,;$$

$$\therefore n = \frac{0{\cdot}3010300}{0{\cdot}0113943} = 26{\cdot}4 \text{ years.}$$

BIRTH-RATES AND DEATH-RATES.

Birth-rates and death-rates are reckoned as so many births and deaths per 1,000 inhabitants living, at all ages.

Example :

There were 243 births in a year amongst a population of 9,500 (estimated to the middle of the year) ; find the annual birth-rate for the year.

There were 243 births in 9,500 persons,

$$\text{or } \frac{243}{9500} \text{ births per person,}$$

$$\text{or } \frac{243 \times 1000}{9500} = 25{\cdot}6 \text{ births per 1,000 persons ;}$$

therefore the annual birth-rate is 25·6 ;

$$\text{or, generally, annual birth-rate} = \frac{\text{registered births} \times 1{,}000}{\text{population}}.$$

Similarly, the annual death-rate per 1,000

$$= \frac{\text{registered deaths} \times 1{,}000}{\text{population}}.$$

This rate is known as the 'general,' 'recorded,' or 'crude' death-rate.

The **Zymotic Death-rate** is also stated as a rate per 1,000 living, thus :

$$\text{Z.D.R.} = \frac{\text{deaths from zymotic diseases} \times 1{,}000}{\text{population}},$$

the zymotic diseases being small-pox, diphtheria, scarlatina, measles, whooping-cough, diarrhœa, and fever—enteric, typhus, and continued.

Note the effect of wrongly estimating the population. The greater the denominator of a fraction, the less is its value. Therefore, the greater the population in the fraction $\frac{\text{registered births} \times 1{,}000}{\text{population}}$, the less the value of such fraction will be, and consequently the less the birth-rate. Therefore, if a population be over-estimated, the birth-rate and death-rate will each be stated too low, and *vice versâ*.

The **Infantile Mortality Rate** is stated as an annual rate of so many deaths under 1 year of age, to 1,000 births registered during the year.

Example:

There were 500 births registered during the year, and 70 deaths amongst infants under 1 year of age; find the infantile mortality rate.

There were 70 deaths per 500 births,

or $\frac{70}{500}$ deaths per birth,

or $\frac{70 \times 1000}{500} = 140$ deaths per 1,000 births;

therefore I.M.R. $= 140$;

$$\text{or, generally, I.M.R.} = \frac{\text{deaths under 1 year} \times 1{,}000}{\text{registered births}}.$$

To find the mean annual birth-rate for a period of 10 years.

Estimate the population for each of the 10 years, add them together, and divide by 10. The result is the 'mean annual population' (or the method shown on p. 136 may be used). Add together the number of births during the 10 years, and divide by 10; this gives the mean number of births per annum.

Then, as in the case of the annual birth-rate,

$$\left.\begin{array}{r}\text{mean annual birth-rate for}\\\text{the period of 10 years}\end{array}\right\} = \frac{\text{mean births} \times 1{,}000}{\text{mean annual population}}.$$

Similarly,

$$\left.\begin{array}{r}\text{mean annual death-rate for}\\\text{period of 10 years}\end{array}\right\} = \frac{\text{mean deaths} \times 1{,}000}{\text{mean annual population}}.$$

Annual Rates for Short Periods.

A 'weekly death-rate' of 21·8 per 1,000 does not mean that 21·8 deaths per 1,000 inhabitants occurred during that week; but that if the deaths that week continue at the same rate throughout the year, they will produce an *annual* death-rate of 21·8 per 1,000.

A year consists of 365 days, 5 hours, 48 minutes, 57 seconds.

i.e., 1 year = 365·24226 days,
= 52·17747 weeks;
a quarter = from 90 to 92 days;
a month = from 28 to 31 days.

The following general formula may be used for ascertaining annual rates from (*a*) weekly, (*b*) monthly, and (*c*) quarterly returns.

(*a*) **Weekly**:

Let b = births in a week;
x = number of days in a week (viz., 7).

Then b births in x days $= \frac{b}{x}$ births per diem,

and $\frac{b \times 365{\cdot}24226}{x}$ births per annum,

and $\frac{b \times 365{\cdot}24226 \times 1{,}000}{x \times \text{population}}$ births per 1,000 per annum.

In the case of monthly and quarterly returns, the above formula will apply, with the following alterations:

(*b*) **Monthly:**

Let b = number of births in the month,
x = number of days in the month (28 to 31).

(*c*) **Quarterly**:

Let b = number of births in the quarter,
x = number of days in the quarter (90 to 92).

And similarly for death-rates.

Example:

In a population of 101,786 there were 213 deaths in 5 weeks; what was the death-rate per 1,000 per annum?*

213 deaths in 5 weeks = 42·6 deaths per week;

$$\frac{42{\cdot}6 \times 365{\cdot}24226 \times 1000}{7 \times 101786}, \left(\text{or, } \frac{42{\cdot}6 \times 52{\cdot}17747 \times 1000}{101786}\right)$$

$$= 21{\cdot}8 \text{ per } 1{,}000.$$

To find the death-rate of a combined district, where the death-rates of the individual districts are known.

Let A = population in one district,
and x = its death-rate per 1,000,
then $\frac{x \times A}{1000}$ = total deaths in A.

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* D.P.H. Exam. (Roy. Coll. Phys. Surg.).

Let B = population in second district,
and y = its death-rate per 1,000,
then $\frac{y \times B}{1000}$ = total deaths in B;

$\therefore \frac{x \times A}{1000} + \frac{y \times B}{1000} = \frac{Ax + By}{1000}$ = total deaths in combined district (A + B),

and $\frac{Ax + By}{1000(A + B)}$ = death-rate per unit in A + B,

and $\frac{Ax + By}{A + B}$ = death-rate per 1,000 in A + B.

This expression furnishes an example of what is known as a *weighted average*, where each 'quantity' is multiplied by its 'weight' (*i.e.*, the number of persons or things connected with it), the sum of the products forming the numerator, and the sum of the weights the denominator.

In this example the 'quantities' are the death-rates, and the 'weights' the populations.

Example:

One district of a town has a population of 150,000, with a death rate of 21 per 1,000 per annum, and another district of the same town has a population of 20,000, with a death-rate of only 15. Show by calculation what is the death-rate of the combined districts.*

$$\frac{Ax + By}{A + B} = \frac{(150{,}000 \times 21) + (20{,}000 \times 15)}{150{,}000 + 20{,}000} = 20{\cdot}3 \text{ per } 1{,}000.$$

That this is the correct death-rate for the combined district may be readily proved; for a death-rate of 21 per 1,000 in one district means 3,150 deaths in the 150,000; and one of 15 per 1,000 in the second district means 300 deaths in the 20,000, or a total of 3,150 + 300 deaths in the 150,000 + 20,000 inhabitants—that is, 3,450 deaths in 170,000, or 20·3 per 1,000.

* D.P.H. Exam. (Roy. Coll. Phys. Surg.).

It would have been an error to assume that the death-rate of the combined district was the average of the death-rates of the individual districts—viz., $\frac{21+15}{2}=18$. This is only true when the populations of the individual districts are equal, for then A = B, and the expression $\frac{Ax+By}{A+B}$ becomes $\frac{x+y}{2}$.

The death-rates 21 and 15 per 1,000 are simply *averages*, and 'when an average is deduced from two or more averages—that is, when an average of averages is taken—there must be the same number of numerical units in each' (Parkes).

If the populations of A and B had been equal—*i.e.*, had contained the same number of units—then the average of 21 and 15—viz., 18—would be the correct death-rate for the combined district, but not otherwise.

The error arising from 'averaging an average' is well shown in the following :

If a man walk and run alternate miles at the respective rates of 4 and 12 miles an hour, his average travelling is 6 miles an hour, not 8 (the mean of 4 and 12, which numbers are only 'averages' themselves). For he walks the first mile in 15 minutes and runs the second mile in 5 minutes ; he therefore covers 2 miles in 20 minutes. That is, his rate of travelling averages 6 miles an hour.

If the combined district consists of three parts, C being the population of the third part, and z its death-rate per 1,000, then $\frac{Ax+By+Cz}{A+B+C}$ = death-rate of combined district.

Example :

One part of a town has a population of 6,000, and a death-rate of 19; a second part has a population of 12,000, and a death-rate of 21; and the remaining part has a population of 9,000, and a death-rate of 25. What is the death-rate of the whole town?*

This may be calculated from the formula just given, where

$$A = 6{,}000, \quad x = 19,$$
$$B = 12{,}000, \quad y = 21,$$
$$C = 9{,}000, \quad z = 25;$$

or independently of the formula, as follows:

A has 19 deaths per 1,000,

or $\frac{19 \times 6000}{1000} = 114$ deaths in the 6,000;

B has 21 deaths per 1,000,

or $\frac{21 \times 12000}{1000} = 252$ in the 12,000;

C has 25 deaths per 1,000,

or $\frac{25 \times 9000}{1000} = 225$ in the 9,000;

that is, (114+252+225) deaths in a total population of (6,000+12,000+9,000), or 591 deaths in 27,000;

that is, $\frac{591 \times 1000}{27000} = 21{\cdot}8$ per 1,000.

The same formula may be used when the population and death-rate of one part of the district and the whole district are given, and it is required to find the death-rate of the other part.

* D.P.H. Exam. (Roy. Coll. Phys. Surg.).

Example:

The population of a combined district is 14,000, and its death-rate 13 per 1,000; the population of one part is 6,000, and its death-rate 10 per 1,000; find the death-rate of the other part.

$$x=10,\ A=6{,}000,\ A+B=14{,}000;$$

$$\therefore B=8{,}000. \quad \text{Find } y.$$

$$\frac{Ax+By}{A+B}=13;\ \therefore Ax+By=13(A+B)$$

$$\therefore y=\frac{13(A+B)-Ax}{B}=\frac{(13\times 14000)-(6000\times 10)}{8000}=15{\cdot}25.$$

Corrected Death-Rates.

Example:

What do you understand by 'recorded,' 'standard,' and 'corrected' death-rates? How are they obtained?*

In comparing the death-rates of two or more towns, the age and sex-distribution of their respective populations must be considered, for the following reasons:

Under five and over fifty-five years of age, the death-rate is higher than the combined rate for all ages, whilst between five and fifty-five it is lower.

An undue proportion of infants or old people will thus raise the death-rate, independently of all other considerations.

The following example will tend to emphasise this point:

A and B are two towns, having the same numerical population, viz., 100,000, each of which may be divided

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* D.P.H. Exam. (Roy. Coll. Phys. Surg.).

into two groups—(*a*) those under 5 years of age, and (*b*) those over 5.

	A		B	
	Death-Rate per 1,000	Population	Death-Rate per 1,000	Population
(*a*) Under 5 (*b*) Over 5	24·5 14·5	10,000 90,000	25 15	3,000 97,000
		100,000		100,000
Death-rate for whole town	15·5		15·3	

The table shows that A has the lower death-rate in each group; it might be assumed, therefore, that it would also have a lower combined death-rate than B. But in A 10 per cent. of the population are under 5, whilst in B only 3 per cent. are under 5; and if the general death-rate for each town be calculated (as shown on p. 139), it will be found that A's death rate = 15·5, whilst B's = 15·3. It is evident that the higher death-rate in A is due *solely* to the larger proportion of young children contained therein.

Again, at nearly every age-group the death-rate of females is lower than that of males; an excess of females, therefore, in a population will lower the death-rate. Hence, some correction must be made in the death-rate of a town for any such disproportionate distribution of age and sex, before its rate is comparable with that of another town. The method of making this correction is as follows:

Age-period.	*Population, divided for Age and Sex (1901 Census).*		*Death-rate (E. and W.) (1891-1900.)*		*'Expected' Number of Deaths.*	
	M.	*F.*	*M.*	*F.*	*M.*	*F.*
0– 5	5,622	5,713	62·7	52·8	352·5	301·6
5–10	5,345	5,471	4·3	4·4	22·9	24·0
10–15	5,134	5,194	2·4	2·6	12·3	13·5
15–20	4,961	5,049	3·8	3·7	18·8	19·7
20–25	etc.	etc.	etc.	etc.	etc.	etc.
25–35						
35–45						
45–55						
55–65						
65–75						
75–85						
85–						
Totals	48,761	51,239			837	909
	100,000				1,746	

Consider the case of a town which at the last census showed a population of 100,000 inhabitants. Divide up the population into the twelve age-groups, as in the accompanying table, according to the figures obtained from the last census returns, distinguishing the sexes. Apply to the population at each age-group and each sex the death-rate for that particular age and sex which obtained for England and Wales generally during the last intercensal period of ten years, and calculate the number of deaths—'expected deaths'—which each such rate produces, thus:

Take the males at ages 0–5—viz., 5,622. The E. and W. death-rate for males at that age is 62·7 per 1,000, and this rate would produce $\frac{62·7 \times 5622}{1000} = 352·5$ deaths amongst the 5,622. Again, between 5–10 years, a rate of 4·3 per 1,000 means $\frac{4·3 \times 5345}{1000} = 22·9$ deaths amongst the 5,345 male population at that age-group.

Proceed similarly for each age-group and each sex, when the total deaths thus calculated will be found to be (say) 1,746. In other words, if the deaths amongst the 100,000 inhabitants had occurred at the same rate as obtained in E. and W. generally, there would have been 1,746 deaths, or $\frac{1746 \times 1000}{100000} = 17·46$ per 1,000.

This rate—viz., 17·46 per 1,000—is known as the **standard death-rate.** [The standard death-rate is thus seen to be merely an hypothetical one, calculated on the assumption that the deaths in the town occur at the same rate as in E. and W. generally.]

Having applied the E. and W. death-rate at each group to the population of the town, the standard death-rate of the town thus obtained ought to be the same as in E. and W., *other things being equal.* But this is not the case, the death-rate for E. and W. during the ten years 1891-1900 having been 18·2; therefore, the age and sex distribution of the town are obviously different from that of E. and W. The standard death-rate must, therefore, be raised in the proportion of—

$$\frac{\text{rate for E. and W.}}{\text{standard death-rate}} = \frac{18·2}{17·46} = 1·0423,$$

to make it comparable with that of E. and W.

The **factor for correction**, then, is 1·0423; and if the recorded (*i.e.*, the general, or crude) death-rate in any year be multiplied by this factor, the **corrected death-rate** is obtained; or,

$$\left\{\begin{array}{c}\text{corrected}\\ \text{death-rate}\end{array}\right\} = \left\{\begin{array}{c}\text{recorded}\\ \text{death-rate}\end{array}\right\} \times \text{factor.}$$

If, for example, the recorded death-rate of the town for 1905 was 16·2, then the corrected death-rate for that year would be $16{\cdot}2 \times 1{\cdot}0423 = 16{\cdot}9$.

Age and sex being thus corrected for, any difference in the rates of the two towns compared may be put down to influences appertaining to the place.

Unless the industrial character of the town undergoes some important change, it is found that the age and sex distribution, expressed as a ratio to the total population, remain fairly constant during an intercensal period. For this reason, the 'factor' is only calculated once every ten years, the last factor holding good until the next census provides new figures from which a new factor may be formed.

A Standard Million.

It has been seen that the standard death-rate is obtained by applying the E. and W. death-rate to the population of the town, divided into sex and age-groups.

The following is an alternative method of correcting the death-rate, thereby eliminating the influences of varying age and sex distribution.

The census returns for 1911 showed that the total population of England and Wales was 36,070,492, and that the males under 5 numbered 1,896,041. How many males under 5 would there have been, if the total population had been 1,000 000?

$$1{,}896{,}041 \times \frac{1{,}000{,}000}{36{,}070{,}492} = 52{,}564.$$

Similarly the males between 5-10 were stated to be 1,852,192; therefore in a population of 1,000,000 they would have been

$$1{,}852{,}192 \times \frac{1{,}000{,}000}{36{,}070{,}492} = 51{,}349.$$

By making a similar calculation for each age-group and sex, a table may be constructed, which shows what is termed a **standard million.** And since the age and sex distribution in this million is exactly proportional to that of England and Wales, the table may be considered to represent a 'miniature England and Wales' reduced in scale approximately $\frac{1}{36}$.

Next, apply to the population at each age-group and sex of this million the respective death-rate, for each group, found to exist in the town under consideration, and thus estimate what would have been the general death-rate in England and Wales had each age-group of its population been subject to the same mortality as that which prevailed in this town.

Thus, of the two methods of correcting the death-rate the former (p. 149) applies the England and Wales death-rate to the population of the town, whilst the latter method applies the death-rate of the town to a miniature population of England and Wales.

Comparative Mortality Figure.

Example:

Explain fully what is meant by the term 'Comparative Mortality Figure' as used by the Registrar-General in connection with (*a*) general mortality, (*b*) mortality from occupation.*

(*a*) *General Mortality:*

The corrected death-rate referred to on p. 150 may be

* D.P.H. Exam. (Roy. Coll. Phys. Surg.).

stated in another manner: If the death-rate for E. and W. for the same year (1905) be represented by the number 1,000, what number will represent the corrected death-rate of the town?

$$\left\{\begin{matrix}\text{death-rate}\\ \text{for E. and W.}\end{matrix}\right\} : \left\{\begin{matrix}\text{corrected death-}\\ \text{rate for town}\end{matrix}\right\} :: 1{,}000 : x$$

$$\therefore x = \frac{\text{corrected death-rate} \times 1{,}000}{\text{death-rate for E. and W.}}.$$

The rate for E. and W. in 1905 was 15·2;

$$\therefore x = \frac{16{\cdot}9 \times 1{,}000}{15{\cdot}2} = 1{,}112.$$

This number—1,112—is the **comparative mortality figure**, and is the corrected death-rate for the town compared with the death-rate of E. and W., taken as 1,000. It means that the same number of persons which in 1905 produced 1,000 deaths in E. and W. produced 1,112 deaths in this particular town, after correction for differences in age and sex distribution.

[NOTE.—In constructing the 'factor,' the death-rate for E. and W. which is made use of is the annual rate *for the past ten years.* In estimating the C.M.F. for any year, the E. and W. death-rate *for that particular year only* is used, for purposes of comparison.]

(*b*) *Mortality from occupation:*

The C.M.F. has another use in statistics, in addition to the one already described, since it is a means by which the healthiness of different occupations may be compared with one another.

For this purpose, males between the working ages of 25 and 65 only are considered.

Tatham has shown, from the statistics for 1900-1902, that the death-rate of all males between 25 and 65 years of age is 14·08 per 1,000, and 14·08 deaths per 1,000 persons mean 1,000 deaths in $\frac{1,000 \times 1,000}{14 \cdot 08} = 71,005$ persons. The number 71,005, then, forms a **standard population**, meaning thereby the number of males between 25 and 65 in E. and W., amongst whom 1,000 deaths occur annually. This standard population is then divided up into four age-groups, which, approximately, are as follows:

Age.			*Thousands.*
25–35	...	...	26
35–45	...	...	20½
45–55	...	...	15
55–65	...	...	9½
			71

Take, for example, the cutlery industry. From the census returns, the number of cutlers living at each of the four age-periods is obtained, and also the number of deaths which occur amongst these in each period. From these data the death-rate per 1,000 at each age-group is calculated. These death-rates are then applied, respectively, to each of the four age-groups in a 'standard population' of cutlers (in a manner similar to that employed on p. 148). Thus, the number of deaths that would occur amongst 26,000 cutlers aged 25-35 is calculated, if the corresponding rate of mortality occurring in the cutlery trade at this age be applied to them. Similarly, the deaths are calculated which would occur amongst 20,500 cutlers aged 35-45, amongst 15,000 aged 45-55, and amongst 9,500 aged 55-65.

The calculated deaths in these four groups are added together, and their sum is found to be (say) 1,460. The C.M.F. for the cutlery industry, therefore, is 1,460, which means that amongst a certain number of cutlers, 1,460 deaths occur, whilst amongst the same number of 'males at working ages,' only 1,000 deaths would occur.

POISSON'S RULE.

If statistics are to have any value, it is essential that the number of the units from which they are compiled shall be sufficiently large.

If M = number of cases,
m = number that recover,
n = number that die,

then $\frac{m}{M}$ = proportion of recoveries, and

$\frac{n}{M}$ = proportion of deaths.

Now, if it be assumed that in a second series of cases the fractions $\frac{m}{M}$ and $\frac{n}{M}$ will have the same value as in the first series, the error fallen into will vary, according to **Poisson's rule**, between $+2\sqrt{\frac{2mn}{M^3}}$ and $-2\sqrt{\frac{2mn}{M^3}}$. That is, the fraction $\frac{m}{M}$ in the first series will, in the second, lie somewhere between $\left(\frac{m}{M}+2\sqrt{\frac{2mn}{M^3}}\right)$ and $\left(\frac{m}{M}-2\sqrt{\frac{2mn}{M^3}}\right)$.

Now, the greater the denominator, the less the value of a fraction. Therefore, the greater M^3 is, the less will be the value of $2\sqrt{\frac{2mn}{M^3}}$. That is, the greater the number of recorded cases, the less will be the error when applying the statistics to subsequent similar series.

If $M = 10$ cases,
$m = 8$ that recover,
$n = 2$ that die,

then the possible error is $2\sqrt{\frac{2mn}{M^3}} = 2\sqrt{\frac{2.8.2}{(10)^3}} = 2\sqrt{\frac{32}{1000}} = 2\sqrt{0\cdot032} = 0\cdot3576$ to unity, or 35·76 per cent.

The cases that die are 2 in 10, or 20 per cent. Therefore, in a second series of cases they may be either $20 + 35\cdot76 = 55\cdot76$ per cent., or $20 - 35\cdot76 = -15\cdot76$ per cent.

Similarly, the recoveries are 8 in 10, or 80 per cent.; therefore, in a second series they may be either

$$80 + 35\cdot76 = 115\cdot76 \text{ per cent., or}$$
$$80 - 35\cdot76 = 44\cdot24 \text{ per cent.}$$

That is to say, the mortality may be nearly 16 per cent. less than nothing (−15·76), or the recoveries may be nearly 16 per cent. greater than the total number of cases (115·76), which obviously cannot be.

If another similar, but much larger, series of cases be taken, in which

$M = 100{,}000$,
$m = 80{,}000$,
$n = 20{,}000$,

the possible error will be

$$2\sqrt{\frac{2mn}{M^3}} = 0\cdot003576 \text{ to unity, or } 0\cdot3576 \text{ per cent.}$$

The cases that die are 20 per cent.; therefore, in a second series they may be either

$$20 + 0\cdot3576 = 20\cdot3576 \text{ per cent., or}$$
$$20 - 0\cdot3576 = 19\cdot6424 \text{ per cent.}$$

It is thus seen how small the error is, where a large number of cases is under consideration.

To find the Relative Values of Two or More Series of Observations.

If the figures in the two previous examples be adopted, then

error in first series = 35·76 per cent.,
" second " = 0·3576 " "

$$0{\cdot}3576 : 35{\cdot}76 :: 1 : 100,$$
$$:: \sqrt{(1)^2} : \sqrt{(100)^2},$$
$$:: \sqrt{1} : \sqrt{10{,}000},$$
$$:: \sqrt{10} : \sqrt{100{,}000},$$

and in the first series there were 10 cases,
and in the second series there were 100,000 cases.

Therefore,

$$\left\{\begin{array}{c}\text{error in}\\ \text{second}\\ \text{series}\end{array}\right\} : \left\{\begin{array}{c}\text{error in}\\ \text{first}\\ \text{series}\end{array}\right\} :: \left\{\begin{array}{c}\text{square root}\\ \text{of number}\\ \text{of cases}\\ \text{in first}\\ \text{series}\end{array}\right\} : \left\{\begin{array}{c}\text{square root}\\ \text{of number}\\ \text{of cases in}\\ \text{second}\\ \text{series.}\end{array}\right\}$$

That is,

the error varies inversely as the square root of the number of cases; and as the value will vary inversely as the error, it follows that the values of two series vary directly as the square roots of the number of cases in the respective series.

Example:

In 100 cases, 70 recover, and 30 die. Find the error in the similar, but larger, series of 10,000 cases.

Error in first series $=2\sqrt{\frac{2mn}{M^3}}=2\sqrt{\frac{2.70.30}{(100)^3}}=0{\cdot}13$ to unity, or 13 per cent.

In second series, let $x=$ error,

$$\text{then } x : 0{\cdot}13 :: \sqrt{100} : \sqrt{10{,}000};$$

$$\therefore x=\frac{0{\cdot}13\times\sqrt{100}}{\sqrt{10{,}000}}=0{\cdot}013 \text{ to unity, or } 1{\cdot}3 \text{ per cent.}$$

From the first series it is seen that recoveries may vary between $70+13=83$, and $70-13=57$; but in the case of the second series, only between $70+1{\cdot}3=71{\cdot}3$, and $70-1{\cdot}3=68{\cdot}7$.

The respective values of the two series are as

$$\sqrt{100} : \sqrt{10{,}000}=1 : 10.$$

Averages and Probable Error.

The arithmetical mean is the one most frequently made use of in statistical enquiries, and is obtained as follows:

Take the sum of all the numerical values, and divide this by the number of items in the series.

Thus, in the series 2, 3, 4, 6, 9, 11, 14, the sum of these figures is 49, and the number of items composing the series is 7. The arithmetical mean, or average, therefore is $\frac{49}{7}=7$.

The approximation of this average to the truth may be ascertained by finding the *probable error*, as follows:

(i.) Take the mean of the series.

(ii.) Find the mean of all the observations above the mean, and subtract the mean from it; the difference is the *mean error in excess.*

(iii.) Find the mean of all the observations below the mean, and subtract this from the mean; the difference is the *mean error in deficiency.*

(iv.) Take the mean of (ii.) and (iii.), which gives the *mean error.*

(v.) The *probable error* is $\frac{2}{3}$ of the mean error.

Thus, in the series given above, the mean is $\frac{49}{7}=7$. The observations above the mean—that is, above 7—are 9, 11, and 14, and their mean is $\frac{9+11+14}{3}=11{\cdot}33$; the mean error in excess is therefore $11{\cdot}33-7=4{\cdot}33$. The mean of the observations below 7 is $\frac{2+3+4+6}{4}=3{\cdot}75$, and the mean error in deficiency is $7-3{\cdot}75=3{\cdot}25$.

The mean error will be $\frac{4{\cdot}33+3{\cdot}25}{2}=3{\cdot}79$, and the probable error $\frac{2}{3}\times 3{\cdot}79=2{\cdot}53$.

If another series be taken, *e.g.*,

2, 3, 5, 19, 29, 43, 81,

the probable error may be similarly estimated, and will be found to be 14·58.

The relative values of these two series will vary inversely as the squares of the probable errors; that is,

$$\left.\begin{matrix}\text{Value of}\\ \text{1st series}\end{matrix}\right\} : \left\{\begin{matrix}\text{Value of}\\ \text{2nd series}\end{matrix}\right\} :: (14{\cdot}58)^2 : (2{\cdot}53)^2$$

$$:: 33 : 1.$$

The value of the 1st series is thus 33 times greater than that of the 2nd.

The 'error' shows how far one is justified in assuming that the same proportions will be repeated in future cases. It will have been gathered that the word 'error' is not synonymous with 'mistake,' but expresses the difference between an estimate and an exact measurement.

CHAPTER XII

LIFE-TABLES

Example:

How is a life-table constructed, and what are its uses?*

Probability:

If an event can happen in a ways and fail in b ways, and all these ways are equally likely to occur, then the probability of its happening is $\frac{a}{a+b}$, and the probability of its failing is $\frac{b}{a+b}$. That is, the probability of an event happening is expressed by a fraction whose numerator is the number representing the number of favourable events, and whose denominator is the total number of possible events, favourable or unfavourable. [Events are *mutually exclusive* when the supposition that any one takes place is incompatible with the supposition that any other takes place.] When different events are mutually exclusive, the chance that one or other of the different events will occur is the sum of the chances of the separate events. Therefore the chances of the event either happening or failing will be $\frac{a}{a+b}+\frac{b}{a+b}$.

But the sum of these two fractions $=1$; and as it is certain that the event will either happen or fail, the

* D.P.H. Exam. (Roy. Coll. Phys. Surg.).

probability of a certainty is unity. Two probabilities which together make up unity are called 'complementary probabilities.'

If p_x = probability of a person, aged x, surviving a full year, and

q_x = probability of the person, aged x, dying within the year,

then $p_x + q_x = 1$, and $q_x = 1 - p_x$.

Example:

If there are 1,000 persons aged x, of whom 900 survive to the age $x+1$, what is the probability of any person aged x living a year?

Each of the 1,000 persons must either live or die; therefore the total number of possible events = 1,000. Since 900 survive the year, the number of favourable events is 900; therefore,

$$p_x = \frac{900}{1000} = \frac{9}{10} = 0{\cdot}9,$$

$$\text{and } q_x = 1 - \frac{9}{10} = \frac{1}{10} = 0{\cdot}1;$$

$$\text{and since } p_x = \frac{900}{1000},$$

$$\therefore p_x = \frac{\text{number living at end of year}}{\text{number living at beginning of year}}.$$

Again,

Let P_x* = mean (or central) population at age x—*i.e.*, between the ages x and $x+1$;

d_x = the number of deaths which occur at age x;

m_x = the mean (or central) death-rate per unit of population at age x;

then, d_x deaths amongst P_x persons mean $\frac{d_x}{P_x}$ deaths per unit of population;

$$\text{that is, } \frac{d_x}{P_x} = m_x.$$

* Dr. Farr's notation is retained here; actuaries use the symbol 'L_x.'

Now these deaths are assumed to be evenly distributed throughout the year—some of those dying having just reached the age x, and others being nearly $x+1$ years—so that half of them, $\frac{d_x}{2}$, occur in the earlier part of the year, and $\frac{d_x}{2}$ in the later part; and since P_x = pop. at centre of year of age—*i.e.*, after half the deaths have occurred—the population at the beginning of the year may be represented by $P_x+\frac{d_x}{2}$, and that at the end of the year by $P_x-\frac{d_x}{2}$; that is,

$$\frac{\text{no. living at end of year}}{\text{no. living at beginning of year}}=\frac{P_x-\frac{d_x}{2}}{P_x+\frac{d_x}{2}}=\frac{\text{pop.}-\frac{1}{2}\text{ deaths}}{\text{pop.}+\frac{1}{2}\text{ deaths}}.$$

$$\text{But, }\frac{P_x-\frac{d_x}{2}}{P_x+\frac{d_x}{2}}=\frac{2P_x-d_x}{2P_x+d_x}=\frac{2-\frac{d_x}{P_x}}{2+\frac{d_x}{P_x}}=\frac{2-m_x}{2+m_x}.$$

There are thus three ways of expressing p_x—viz.,

(i.) $$p_x=\frac{\text{no. living at end of year}}{\text{no. living at beginning of year}};$$

(ii.) $$p_x=\frac{\text{pop.}-\frac{1}{2}\text{ deaths}}{\text{pop.}+\frac{1}{2}\text{ deaths}};$$

(iii.) $$p_x=\frac{2-m_x}{2+m_x}.$$

Construction of a Short Life-Table by Dr. Farr's Method.

The statistics regarding population, etc., which are available for the construction of an ordinary life-table, are those extracted from the census returns, and issued by the Registrar-General. These statistical tables give the enumerated populations at each year of age up to five years, and afterwards in groups of five or ten years only. A *full* life-table gives the expectation of life at each year of age, and for its construction it is necessary either (i.) to determine the population living *in each year of age* from the figures *for each group of ages*, as given in the census returns; or (ii.) to calculate the probability of living 1 year (p_x) for each group of ages, and from these grouped probabilities deduce the similar probabilities for each individual age. Either procedure requires a knowledge of 'finite differences,' or of 'graphic interpolation,' and need not be discussed here.

Dr. Farr (in the supplement to the thirty-fifth annual report of the Registrar-General) introduced a 'short' method of constructing a life-table, wherein the expectation of life is only calculated for the groups of years which correspond with those of the census returns, no interpolation being required.

For local life-tables this 'short' method gives sufficiently accurate results for the greater part of life. Towards the end of the table its results are unreliable, the expectation given being too high. Dr. Hayward, however, has devised one or two modifications in the short method, whereby its results may be made to approximate very remarkably to those of the extended method. For these the reader is referred to his original paper—'On Life-Tables: Their Construction and Practical Application'—published in vol. lxii. of the *Journal of the Royal Statistical Society*. They will be briefly referred to later on, after the ordinary construction of the short table, as devised by Dr. Farr, has been described in detail.

Data.—For a life-table constructed on the returns for the ten years 1901-1910 the following data are necessary:

(i.) The populations at the censuses of 1901 and 1911, divided according to age and sex.

(ii.) The number of deaths in the ten years 1901-1910, distinguishing age and sex.

These data (for males only) are set out in Tables I. and II. (p. 164). To these may be added—

(iii.) The number of births in the ten years, distinguishing the sexes. (This item is required for deciding on the 'radix,' as explained later.)

A separate life-table is constructed for each sex. Only the Males Table will be described here; but it must be understood that every process and calculation shown below must be repeated, substituting the figures and data applying to females for those already used in the case of males. Each age-group must also be treated separately, as an individual population.

Preliminary Calculations.—Before commencing the construction of the life-table some preliminary calculations are required, having for their object the furnishing of the requisite data for the formation of the p_x column. This column, although frequently not inserted in a life-table, is, in fact, the foundation of the table. These calculations involve the finding of the total populations for the ten years at each age-group—that is, the total number of lives between these ages, subjected to a year's risk during these ten years. From the total populations thus found the mean annual populations are known.

Total Lives at Risk, and Mean Annual Populations.—Each age-group must be considered separately. If, for example, the ages 25-35 be taken, it will be seen from Table I. that between these ages there were 4,647

Age.	*Table I.* Census Returns (Males only). 1901.	1911.	*Table II.* Male Deaths in 10 Years (1901-1910).	*Table III.* Total Male Population for 10 Years.	*Table IV.* p_x
0–1	713	710	1,245	7,109	·83896
1–2	605	645	368	6,240	·94270
2–3	674	587	152	6,314	·97618
3–4	630	621	97	6,250	·98461
4–5	603	588	68	5,948	·98858
5–10	2,938	2,941	126	29,398	·99570
10–15	2,622	2,800	65	27,162	·99761
15–20	2,601	2,785	103	27,007	·99619
20–25	2,673	3,349	152	29,824	·99491
25–35	4,647	5,811	352	51,814	·99323
35–45	3,308	4,529	444	38,577	·98856
45–55	2,347	3,066	508	26,735	·98117
55–65	1,391	1,890	566	16,158	·96557
65–75	758	970	601	8,530	·93194
75–85	246	346	425	2,911	·86393
85–	21	50	95	331	·74901
Total	26,777	31,688			

The various age-groups read as follows:

0–1 = age under 1;
5–10 = age 5 and under 10, etc.

Table V.	
Total Births in the 10 *Years* 1901-1910.	
Males	9,286
Females	8,964
Total	18,250

living at the 1901 census, and 5,811 living at the 1911 census. It is required to find the total population living between the ages 25-35 during the ten years. This is calculated from the formula shown on p. 137, where this particular example will be found fully worked out, and from which it will be seen that the total population for the ten years at this age-group = 51,814. This calculation must be repeated for each age-group, and the results entered in Table III.

p_x Column.

(Where p_x = probability that a person aged x will survive one year.)

Take the same age-group again—viz., 25-35. The total population for the ten years is 51,814 (Table III.); therefore the mean annual population is 5181·4; the total number of deaths for the ten years is 352 (Table II.); therefore, the mean annual number of deaths is 35·2, the half of which is 17·6; and since—

$$p_x = \frac{\text{pop.} - \frac{1}{2}\text{ deaths}}{\text{pop.} + \frac{1}{2}\text{ deaths}}; \quad \therefore p_{25} = \frac{5181{\cdot}4 - 17{\cdot}6}{5181{\cdot}4 + 17{\cdot}6} = \frac{5163{\cdot}8}{5199} = 0{\cdot}99323.$$

This process must be repeated for each age-group, and the results entered in Table IV.

[*Alternative Method of finding p_x.*

If, in the data provided, the mean annual death-rate per 1,000 at each age-group for the 10 years be given, instead of the actual number of deaths, the formula $p_x = \frac{2 - m_x}{2 + m_x}$ (p. 161) may be used instead of $p_x = \frac{\text{pop.} - \frac{1}{2}\text{ deaths}}{\text{pop.} + \frac{1}{2}\text{ deaths}}$, thus saving the necessity of calculating out the deaths. Thus, if in the age-group 25-35 the death-rate had been stated as a mean annual death-rate of $\left(\frac{35\cdot2 \times 1000}{5181\cdot4} = \right)$ 6·79 per 1,000 for the 10 years, then 0·00679 = death-rate per unit, and $m_x = 0\cdot00679$.

$$\therefore p_x = \frac{2 - m_x}{2 + m_x} = \frac{2 - 0\cdot00679}{2 + 0\cdot00679} = \frac{1\cdot99321}{2\cdot00679} = 0\cdot99323,$$

thus obtaining the same result as by the other method.]

The data are now complete for the

Construction of the Life-Table.

l_x Column.

The first column to be constructed is the l_x column. l_x = the number living who attain the exact age x, whilst l_{x+1} will be the number who survive to the end of the year, since it is the number who reach the next year of age—viz., $x+1$; and since

$$p_x = \frac{\text{no. living at end of year}}{\text{no. living at beginning of year}},$$

$$\therefore p_x = \frac{l_{x+1}}{l_x};$$

$$\therefore l_x \times p_x = l_{x+1}.$$

That is, if the number living at the age x be multiplied by the probability of surviving one year from the age x, the product will give the number living at the next year. Hence :

$$l_0 \times p_0 = l_1$$

$$l_1 \times p_1 = l_2$$

and $l_4 \times p_4 = l_5$.

It is quite immaterial what number is taken for l_0, since the numbers in this column need not be the absolute numbers living, but only relative numbers. The first value in the l_x column—viz., l_0, and which is known as the 'Radix'—is the number of annual births in the imaginary population, and the succeeding numbers show how many persons out of l_0 born alive complete each year of age. It is usual to take 100,000 births, and divide them into males and females in the proportions found to exist during the ten years under consideration. Thus, the total births during the ten years were 18,250 (Table V.), of which 9,286 were males. If there were 9,286 males out of 18,250 births, how many males would there be out of 100,000 births?

$$18{,}250 : 100{,}000 :: 9{,}286 : x;$$
$$\therefore x = 50{,}884.$$

The Males table will therefore start with 50,884 lives, whilst the companion Females table will start with $100{,}000 - 50{,}884 = 49{,}116$. The two tables together will thus trace the life-history of 100,000 persons from birth.

For this table, therefore,

$$l_0 = 50{,}884;$$

and since

$$l_1 = l_0 \times p_0,$$
$$\therefore l_1 = 50{,}884 \times 0{\cdot}83896 = 42{,}689.$$

Similarly:

$$l_5 = l_4 \times p_4;$$
$$\therefore l_5 = 38{,}680 \times 0{\cdot}98858 = 38{,}238.$$

These numbers, when obtained, are entered in the l_x column of the life-table (p. 172).

Having found l_5, the next step is to find l_{10}.

The number surviving one year from age 5 is $l_5 \times p_5$, and the number out of these $(l_5 \times p_5)$ survivors who survive another year is $(l_5 \times p_5) \times p_5$, or $l_5 \times (p_5)^2$. Similarly, the number who survive 5 years from the age 5 will be $l_5 \times (p_5)^5$; that is,

$$l_{10} = l_5 \times (p_5)^5;$$

that is,

$$l_{10} = 38{,}238 \times (0{\cdot}99570)^5$$
$$= 37{,}423.$$

[These examples are best worked out by logarithms, as follows:

$$l_{10} = 38{,}238 \times (0{\cdot}99570)^5;$$
$$\therefore \log l_{10} = \log 38238 + 5 \log 0{\cdot}99570$$
$$= 4{\cdot}5824952 + (5 \times \bar{1}{\cdot}9981285)$$
$$= 4{\cdot}5824952 + \bar{1}{\cdot}9906425$$
$$= 4{\cdot}5731377$$
$$= \log 37423.]$$

Similarly, for the age-groups comprising ten years,

$$l_{25} \times (p_{25})^{10} = l_{35};$$
$$\therefore l_{35} = 35{,}365 \times (0{\cdot}99323)^{10}$$
$$= 33{,}042.$$

[It will be understood that, in reality, p has a different value for each year of age, and in an extended life-table p_5, p_6, ... p_9 are all different. In a short life-table p_5 (or, strictly, p_{5-10}) must be looked upon as an average or mean value of the five probabilities p_5, ... p_9, and is supposed to hold good for each of the five years.]

d_x Column.

(Where d_x = the number of deaths which occur at age x—*i.e.*, between the ages x and $x+1$.)

If from the number living at age x at the beginning of the year, the deaths which occur during the year be subtracted, the difference will give the number living at the beginning of the next year; or,

$$l_x - d_x = l_{x+1},$$
$$i.e., \quad d_x = l_x - l_{x+1}.$$

The d_x column is thus formed as follows:

$$d_0 = l_0 - l_1$$
$$d_1 = l_1 - l_2$$
$$d_5 = l_5 - l_{10}$$
$$d_{25} = l_{25} - l_{35},$$

etc.

For example:

$$d_0 = l_0 - l_1;$$
$$\therefore d_0 = 50,884 - 42,689 = 8,195.$$

And $d_5 = l_5 - l_{10}$;

$$\therefore d_5 = 38,238 - 37,423 = 815,$$

etc.

P_x *Column.*

(Where P_x = mean population at centre of year.)

It has been seen that the number who attain the age $x = l_x$; that is,

l_x = no. reaching the year of age x; whilst

l_{x+1} = no. reaching the end of that year; and since

P_x = mean pop. at centre of year of age, therefore P_x is equivalent to $l_{x+\frac{1}{2}}$; and since the deaths are assumed to be equally distributed throughout the year, the pop. at the middle of the year will be the arithmetical mean between those beginning and those completing that year of age.

$$\text{Hence } P_x = \frac{l_x + l_{x+1}}{2}.$$

For example:

$$P_0 = \frac{l_0 + l_1}{2} = \frac{50,884 + 42,689}{2} = 46,786,$$

$$\text{and } P_3 = \frac{l_3 + l_4}{2} = \frac{39,285 + 38,680}{2} = 38,983.$$

For the age-group 5-10, where 5 years are included, the formula is:

$$P_{5-10}=\frac{l_5+l_{10}}{2}\times 5;$$

$$\therefore P_5=\frac{38,238+37,423}{2}\times 5$$

$$=189,150.$$

And for the age-group where 10 years are included, the formula is:

$$P_{25-35}=\frac{l_{25}+l_{35}}{2}\times 10;$$

$$\therefore P_{25}=\frac{35,365+33,042}{2}\times 10$$

$$\doteqdot 342,035.$$

Having obtained all these values, as set out in the P_x column of the life-table, the Q_x column is next to be formed.

Q_x Column.

This represents the populations at P_x + all higher ages. It thus shows the total number of years lived through by the whole population, from the age x to the end of the table, inclusive. That is:

$$Q_x=P_x+P_{x+1}+P_{x+2}+\ldots\ldots+P_{85}+\ldots\ldots$$

Beginning at the foot of the P_x column, the values of Q_x are obtained by successive additions, thus:

$$P_{85}=\ \ 10,365=Q_{85};$$

$$\text{add } P_{75}=\ \ 52,210;$$

$$\therefore\ \ 62,575=P_{75}+P_{85}=Q_{75};$$

$$\text{add } P_{65}=128,165;$$

$$\therefore\ 190,740=P_{65}+P_{75}+P_{85}=Q_{65},$$

etc.

These numbers—viz., 10,365, 62,575, 190,740, etc., are to be entered from below upwards in the Q_x column of the life-table.

E_x Column.

E_x (also written $\mathring{e}_x$) = the complete expectation of life at the age x. It has been seen that Q_x = the aggregate number of years which persons at the age x will live, from that age and during all succeeding years, to the end of the life-table; and if this total future lifetime be divided equally amongst the survivors at age x—*i.e.*, amongst l_x persons—the quotient will give the average after-lifetime of each individual. That is:

$$\mathring{e}_x = \frac{Q_x}{l_x}.$$

Thus, for the age-group 45-55, $Q_x = 667{,}290$, and $l_x = 29{,}451$;

$$\therefore \mathring{e}_{45} = \frac{667{,}290}{29{,}451} = 22{\cdot}6 \text{ years.}$$

It is thus seen that the expectation of life does not mean the number of years one may reasonably expect to live, but represents the average number of years which persons of a given age in a life-table, taken one with another, actually do live. For this reason, 'average (or mean) after-lifetime' is a better name than 'expectation of life.'

Summary.

The various stages in the construction of a life-table by the short method may be summarized as follows:

Obtain the necessary data.

Deal with each sex separately.

Find the total populations for the ten years at each age-group, and from these the mean annual populations.

From the total deaths in the 10 years find the mean annual number of deaths.

Find p_x for each age-group from the formula

$$p_x = \frac{\text{pop.} - \frac{1}{2}\text{ deaths}}{\text{pop.} + \frac{1}{2}\text{ deaths}}$$

(where 'pop.' = mean annual population for the 10 years, and 'deaths' = mean annual deaths for the 10 years).

Decide on the radix, or value of l_0.

Construct the columns of the life-table by means of the formulæ, and in the order given below:

$$l_x \text{ column}: \; l_x \times p_x = l_{x+1};$$

$$d_x \text{ column}: \; d_x = l_x - l_{x+1};$$

$$\text{P}_x \text{ column}: \; \text{P}_x = \frac{l_x + l_{x+1}}{2};$$

$$\text{Q}_x \text{ column}: \; \text{Q}_x = \text{P}_x + \text{P}_{x+1} + \text{P}_{x+2} + \ldots\ldots;$$

$$\text{E}_x \text{ column}: \; \text{E}_x = \frac{\text{Q}_x}{l_x}.$$

LIFE-TABLE

(FOUNDED ON THE MORTALITY OF THE 10 YEARS 1901-1910).

MALES.

x.	d_x.	l_x	P_x.	Q_x.	E_x.
0–1	8,195	50,884	46,786	2,264,655	44·5
1–2	2,446	42,689	41,466	2,217,869	51·9
2–3	958	40,243	39,764	2,176,403	54·0
3–4	605	39,285	38,983	2,136,639	54·4
4–5	442	38,680	38,459	2,097,656	54·2
5–10	815	38,238	189,150	2,059,197	53·8
10–15	445	37,423	186,005	1,870,047	49·9
15–20	699	36,978	183,142	1,684,042	45·5
20–25	914	36,279	179,110	1,500,900	41·3
25–35	2,323	35,365	342,035	1,321,790	37·3
35–45	3,591	33,042	312,465	979,755	29·6
45–55	5,099	29,451	269,015	667,290	22·6
55–65	7,197	24,352	207,535	398,275	16·3
65–75	8,675	17,155	128,165	190,740	11·1
75–85	6,514	8,478	52,210	62,575	7·3
85–	1,855	1,964	10,365	10,365	5·2
		109			

The reader is reminded that the life-table given above is inserted solely for the purpose of demonstrating the fundamental principles of its construction, and he is specially warned against drawing too strict inferences from it, even with the modifications already referred to on p. 162. The most important of Dr. Hayward's modifications are: (1) The re-estimation of the populations for the first five years of life, the census returns at these ages not being reliable; (2) a modified method of calculating the latter part of the P_x column, whereby the excessive values of P_x towards the end of the table are reduced, and consequently also the excessive values of Q_x and E_x; (3) a method of calculating p_{95}, thus avoiding the manifest error in the method shown above, which often gives to p_{95} a higher value than to p_{85}.

---

e_x, or Curtate Expectation of Life.

Let the sum of the terms $P_x + P_{x+1} + P_{x+2} + \ldots\ldots$ to the end of the table be represented by the symbol ΣP_x (that is, $Q_x = \Sigma P_x$); and let $l_{x+1} + l_{x+2} + l_{x+3} + \ldots\ldots$ be similarly represented by Σl_{x+1}.

In an extended life-table, where all the intermediate values of l are given, Σl_{x+1} will obviously represent the total lives from the beginning of the year of age $x+1$ to the end of the table, whilst ΣP_x will represent the total lives from the middle of the year of age x—or half a year earlier—to the end.

The difference, therefore, between ΣP_x and Σl_{x+1} is half a year's population; and this, when distributed amongst the l_x survivors, gives an average of half a year for each life in the population at that age. That is:

$$\frac{\Sigma P_x}{l_x} - \frac{\Sigma l_{x+1}}{l_x} = \tfrac{1}{2} \text{ year.}$$

$\frac{\Sigma l_{x+1}}{l_x}$, then, gives what is termed the curtate expectation of life at age x, and is represented by e_x;

$$\therefore \frac{\Sigma l_{x+1}}{l_x} = e_x.$$

$$\text{And since } \frac{\Sigma P_x}{l_x} = \frac{Q_x}{l_x} = \mathring{e}_x,$$

$$\therefore \mathring{e}_x - e_x = \tfrac{1}{2} \text{ year,}$$

$$\text{or } \mathring{e}_x = e_x + \tfrac{1}{2}.$$

That is, the curtate expectation of life may be converted into the complete expectation of life by the addition of ½ year.

[This can also be shown as follows:

$$\mathring{e}_x = \frac{Q_x}{l_x} = \frac{\Sigma P_x}{l_x} = \frac{P_x + P_{x+1} + P_{x+2} + \ldots\ldots}{l_x};$$

but $P_x = \frac{l_x + l_{x+1}}{2}$, $P_{x+1} = \frac{l_{x+1} + l_{x+2}}{2}$, etc. (p. 169).

$$\therefore \mathring{e}_x, \text{ or } \frac{\Sigma P_x}{l_x} = \frac{l_x + 2(l_{x+1} + l_{x+2} + \ldots\ldots)}{2\,l_x}$$

$$= \tfrac{1}{2} + \frac{l_{x+1} + l_{x+2} + \ldots\ldots}{l_x}$$

$$= \tfrac{1}{2} + \frac{\Sigma l_{x+1}}{l_x}$$

$$= \tfrac{1}{2} + e_x.]$$

It will thus be seen that the complete expectation of life takes into account the portion of life-time lived in the year in which death occurs—averaged at ½ year—whilst the curtate expectation of life does not.

Probable Life-time (probable duration of life, *vie probable*, equation of life) is the age at which any number of children born will be reduced one-half.

To ascertain the probable life-time, an extended life-table, showing the number living at each age, is required; but the method may be demonstrated by means of the short table.

On reference to the l_x column it will be seen that about the age of 10 years there are approximately 38,000 living, and that these have been reduced to one-half that number about the age 65. Thus the probable life-time at age 10 is (65 − 10 =) 55 years; that is, it is an exactly even chance whether a child of 10 lives a further 55 years or not—*i.e.*, whether he survives the age 65 or dies before.

Below are appended a few examples which can be solved with the aid of a life-table.

Examples:

Find the probability of a man aged 35 surviving 20 years—that is, living until he is 55.

From the l_x column of the life-table it will be seen that there are 33,042 living at 35, and 24,352 living at 55; therefore the probability of his living from 35 to 55 is (p. 159):

$\frac{24{,}352}{33{,}042} = 0{\cdot}7370$, whilst the probability that he will die within the 20 years is $1 - 0{\cdot}7370 = 0{\cdot}263$.

Find the probability of a boy aged 5 surviving to the age 25—that is, surviving 20 years.

The life-table will show that the probability of his surviving is $\frac{35{,}365}{38{,}238} = 0{\cdot}92486$, whilst $1 - 0{\cdot}92486 = 0{\cdot}07514$ is the probability of his dying within that period.

Find the probability of a father aged 35, and a son aged 5, both surviving 20 years.

[The probability that two independent events should both happen, is the product of the separate probabilities of their happening.]

It has been seen that the probability of the man surviving 20 years is 0·7370, whilst the probability of the boy surviving 20 years is 0·92486; therefore the probability of both father and son surviving 20 years is 0·7370 × 0·92486 = 0·68162.

The probability of the father surviving and the son dying during the period is 0·737 × 0·07514 = 0·05538. The probability of the father dying and the son surviving is 0·263 × 0·92486 = 0·24324. The probability of their both dying is 0·263 × 0·07514 = 0·01976; and the probability that both will not die—*i.e.*, that one at least (not specifying which) will survive—is 1 − 0·01976 = 0·98024.

What is the probability that exactly one (not specifying which) will survive the given period?

Probability of father alone surviving = 0·05538, and probability of son alone surviving = 0·24324; and as, by the nature of the question, these events are mutually exclusive (p. 159), the probability that exactly one will survive is, therefore, 0·05538 + 0·24324 = 0·29862. This, of course, is also the probability that exactly one will die.

If, in the above probabilities, the decimal point be moved three places to the right, the chances per 1,000 will be obtained, and may be tabulated as follows:

One at least surviving	980 in 1,000
Son surviving	925 ”

Father surviving	737	in 1,000
Both surviving	681	„
One exactly surviving	298	„
Father dying	263	„
Father dying and son surviving	243	„
Son dying	75	„
Father surviving and son dying	55	„
Father and son both dying ...	20	„

CHAPTER XIII

MENSURATION, ETC.

[ABBREVIATIONS USED: d=diameter, r=radius, l=length, b=breadth, Ch=chord, h=height.]

Linear:

The ratio of the circumference of a circle to its diameter is **constant**, and is usually denoted by the symbol 'π.' Its numerical value, however, cannot be stated exactly. Approximately, it is equal to $\frac{22}{7}$, or, more exactly $\pi = 3{\cdot}1416$.

So we have:

$$\pi = \frac{\text{circumference}}{\text{diameter}} = \frac{\text{circumference}}{2 \times \text{radius}};$$

$$\therefore \text{circumference} = \pi \times 2 \times \text{radius} = 2\pi r.$$

Or it may be stated thus:

$$\pi = \frac{\text{circumference}}{\text{diameter}} = 3{\cdot}1416;$$

$$\therefore \text{circumference} = d \times 3{\cdot}1416.$$

circumference of an ellipse =

$$\pi \times \tfrac{1}{2}\,(\text{major axis} + \text{minor axis}).$$

Superficial Area:

Sectional area of a circle $= \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4}$

$$= \frac{d^2 \times 3{\cdot}1416}{4} = d^2 \times 0{\cdot}7854.$$

Square : $l \times b = l^2$.

Rectangle : $l \times b$.

Parallelogram : length of one side × perpendicular between that side and the one parallel to it.

Triangle : $\frac{1}{2}$ (base × height)—*i.e.*, one-half the parallelogram on same base and of same height.

Triangle (when length of sides only are given) : let a, b, c be the sides, and let $\frac{1}{2}(a+b+c)=s$; then area $= \sqrt{s(s-a)(s-b)(s-c)}$.

Ellipse : major axis × minor axis × 0·7854.

Sphere : $\pi d^2 = 4\pi r^2$.

Segment of circle : $\left(Ch \times h \times \frac{2}{3}\right) + \frac{h^3}{2Ch}$.

Sides of cylinder : $2\pi rh$—*i.e.*, circumference of base (circle) × height.

Curved surface of cone : $\pi r\sqrt{r^2+h^2}$, or

$\frac{1}{2}$ (circumference of base × length of slant side).

Irregular figures bounded by straight lines : divide into triangles, and take the sum of their areas.

Volume, Cubic Capacity, or Solid Contents :

Cube $= l \times b \times h = l^3$.

Rectangle $= l \times b \times h$.

Triangle = area of section of triangle × height.

Cylinder : $\pi r^2 h$ = area of base (circle) × height.

Cone (or pyramid) : $\frac{1}{3} \times \pi r^2 h = \frac{1}{3}$ (area of base × height) $= \frac{1}{3}$ of a cylinder on same base and of same height.

Sphere : $\frac{4}{3}\pi r^3 = \frac{4\pi}{3}\left(\frac{d}{2}\right)^3 = d^3 \times 0{\cdot}5236$.

Dome : $\frac{2}{3} \times \pi r^2 h = \frac{2}{3}$ (area of base × height) $= \frac{2}{3}$ of a cylinder on same base and of same height.

[A triangle—*e.g.*, a prism—has three sides; its base (or section) is triangular, *but its sides are quadrilateral.* A cone, or pyramid, has triangular sides meeting in an apex. A pyramid may have *any number of sides*, therefore its base may be triangular, square, hexagonal, etc. The base of a cone is circular.]

Estimate the cubic capacity of a circular or bell tent, having the following dimensions: The tent is round with vertical sides up to 1 foot high, and then slanting to a central pole 10 feet high; the diameter of the base is 12·5 feet.

In estimating the cubic capacity, the tent should be divided into two parts, the lower part being a cylinder 1 foot high and 12·5 feet in diameter, whilst the upper part is a cone 9 feet high, with a base 12·5 feet in diameter.

Since diameter of cylinder $= 12{\cdot}5$ feet,
$\therefore$ its sect. area $= (12{\cdot}5)^2 \times 0{\cdot}7854 = 122{\cdot}7$ sq. feet,
and since height $= 1$ foot,
$\therefore$ cubic space $= 122{\cdot}7$ cubic feet.

Next, since the cubic space of a cone is $\frac{1}{3}$ that of a cylinder of the same height, and on the same base, find the cubic capacity of a cylinder 9 feet high and 12·5 feet in diameter.

Sect. area of base of cylinder $= 122{\cdot}7$ sq. ft.;
height of cylinder $= 9$ feet;
$\therefore$ cubic capacity of cylinder $= 122{\cdot}7 \times 9 = 1{,}104$ cub. ft.

$$\therefore \text{ cubic capacity of cone} = \frac{1{,}104}{3} = 368 \text{ cub. ft.}$$

Therefore, cubic capacity of tent $= 122{\cdot}7 + 368 = 491$ cub. ft.

For military purposes, this tent usually accommodates about 14 men, thus allowing $\frac{491}{14} = 35$ cub. ft. to each.

Example :

Calculate the cubic capacity of a room 18 feet long, 12 feet broad, and which has a sloping roof 12 feet high on one side and 8 feet high on the opposite side.*

The area of the end wall may be divided into a rectangle 12 ft. × 8 ft., surmounted by a triangle having a base of 12 ft. and a height of 4 ft.

The area of the rectangle is $12 \times 8 = 96$ sq. ft., and that of the triangle $\frac{12 \times 4}{2} = 24$ sq. ft.

$\therefore$ total area of end of room $= 96 + 24 = 120$ sq. ft.

Since the room is 18 ft. long, its cubic capacity will be

$$120 \times 18 = 2{,}160 \text{ cub. ft.}$$

Example :

A circular ward, with a diameter of 28 feet and a dome-shaped roof, the height of whose centre is 17 feet: the height of the dome is 12 feet. Determine the floor-space and the total cubical contents.*

The ward may be divided into two portions, a cylinder of 28 feet diameter and height 5 feet, surmounted by a dome of the same diameter and 12 feet high.

The area of the floor is $(28)^2 \times 0{\cdot}7854 = 615{\cdot}75$ square feet, and the cubical contents of the cylinder $= 615{\cdot}75 \times 5 = 3{,}079$ cubic feet. The cubical contents of the dome will be $\frac{2}{3}$ that of a similar cylinder 12 feet high, or,

$$\tfrac{2}{3}(615{\cdot}75 \times 12) = 4{,}926 \text{ cubic feet.}$$

Total cubical contents of ward $= 3{,}079 + 4{,}926 = 8{,}005$ cubic feet.

---

* D.P.H. Exam., Cambridge.

Example:

According to the Vaccination Order, 1898, a Public Vaccinator must aim at producing 4 separate vesicles, having together a total area of not less than $\frac{1}{2}$ square inch. What should be the minimum diameter of each vesicle?

$$\text{area of 4 vesicles} = \tfrac{1}{2} \text{ square inch;}$$
$$\therefore \text{area of 1 vesicle} = \tfrac{1}{8} \quad \text{,,}$$

It is required to find the diameter of a circle whose area is $\frac{1}{8}$ square inch:

$$\text{area of circle} = d^2 \times 0{\cdot}7854;$$
$$\therefore d^2 \times 0{\cdot}7854 = \tfrac{1}{8}.$$
$$\therefore d^2 = \frac{1}{8 \times 0{\cdot}7854} = \frac{1}{6{\cdot}2832} = 0{\cdot}1591;$$
$$\therefore d = \sqrt{0{\cdot}1591} = 0{\cdot}3989 \text{ inch;}$$

that is, approximately, $\frac{2}{5}$ inch or 10 mm.

Example:

To express the area of a microscope 'field' in terms of the ruled 'squares' of a hæmacytometer.

Adjust the hæmacytometer on the microscope stage in such a position and the draw-tube to such a length, that one of the horizontally-ruled lines of the former exactly coincides with the upper limit of the field, and a second line with its lower limit. These two parallel lines will thus be opposite tangents to the circular field, and the vertical line joining the points where they touch the circle will be a diameter of the field. Count the number of squares in the vertical row adjacent to this diameter, and assume it to be 5. Then the diameter of the field will be the length of 5 sides of a square and the area of the field will be $(5)^2 \times 0{\cdot}7854 = 19{\cdot}635$ squares; therefore

$$2{,}000 \text{ squares} = \frac{2{,}000}{19{\cdot}635} = 102 \text{ fields.}$$

But each square $= \frac{1}{20} \times \frac{1}{20} \times \frac{1}{10} = \frac{1}{4000}$ cubic mm.,

$\therefore$ 4,000 squares = 1 cubic mm.

To enumerate the leucocytes in 1 cubic mm., therefore, it is only necessary to count the number present in 102 fields—*entirely neglecting the ruled lines of the instrument*—and multiply this figure by 2, thus obtaining the number in 4,000 squares. If this again be multiplied by the figure representing the original dilution of the blood (10 or 100, according to the diluting pipette used), the product will give the number of leucocytes in 1 cubic mm. of blood.

Similarly it may be shown that if the diameter of the field be equal to 6, 7, or 8 sides of a square, then 74, 52, or 40 fields respectively will be the equivalent of 2,000 squares.

WEIGHTS AND MEASURES.

Length:

1 metre = 10 decimetres = 100 centimetres = 1,000 millimetres = 39·37 inches = 1·093 yards.

1 inch = 25·3 millimetres.

1 kilometre = 1,000 metres.

1 mile = 1,760 yards = 5,280 feet.

Surface:

144 square inches = 1 square foot.

9 square feet = 1 square yard.

4,840 square yards = 1 acre.

640 acres = 1 square mile.

1 square metre = 10·764 square feet = 1,550 square inches.

Side of a square acre = 69·5 yards.

Capacity :

1,728 cubic inches = 1 cubic foot.

27 cubic feet = 1 cubic yard.

1 litre = 1 cubic decimetre = 1,000 cubic centimetres = 35·3 fluid ounces = 61 cubic inches = 0·22 gallon = 1·7617 pints.

1 gallon = 8 pints = 4·54 litres = 277·274 cubic inches.

1 fluid ounce = 28·4 c.c. = 1·73 cubic inches.

1 cubic foot of water = 1,000 fluid ounces = 6·23 gallons.

1 cubic metre (kilolitre) = 35·3 cubic feet.

Weight :

1 cubic centimetre of distilled water at 4° C. weighs 1 gramme = 1,000 milligrammes.

1 litre of water = 1,000 grammes = 1 kilogramme = 2·2046 pounds.

1 gallon of water = 70,000 grains = 10 pounds (avoir.).

1 cubic foot of water = 62·3 pounds.

1 gramme = 15·432 grains.

1 ounce (avoir.) = 437·5 grains = 28·35 grammes.

1 ounce (troy) = 480 grains.

1 pound (troy) = 5,760 grains.

1 pound (avoir.) = 7,000 grains = 453·6 grammes.

1 minim = 0·91 grain of water.

1 fluid ounce water weighs 1 ounce, or 437·5 grains.

1 ton = 2,240 pounds = 1,016 kilogrammes.

Density :

1 pound per cubic foot = 0·016019 gramme per cubic centimetre.

1 gramme per cubic centimetre = 62·4 pounds per cubic foot.

Atmospheric Pressure :

= 760 millimetres of mercury.

= 29·922 inches of mercury.

= 1·033 kilogrammes per square centimetre.
= 14·73 pounds per square inch.

Atmospheric Pressure and Boiling-point.

Number of Atmospheres.		*Boiling-point.*
1	... ...	100° C.—212° F.
1½	... ...	112°·2 C.—234° F.
2	... ...	120°·6 C.—249° F.
3	... ...	134° C.—273° F.
4	... ...	144° C.—291° F.

TENSIONS OF AQUEOUS VAPOUR.

The following table (by Regnault) gives the tensions of aqueous vapour at different temperatures. For convenience, the corresponding temperatures on the Fahrenheit scale have been added:

C.	*mm.*	*F.*	*C.*	*mm.*	*F.*	*C.*	*mm.*	*F.*
0°	4·600	32·0°	11°	9·792	51·8°	25°	23·550	77°
1	4·940	33·8	12	10·457	53·6	30	31·548	86
2	5·302	35·6	13	11·062	55·4	35	41·827	95
3	5·687	37·4	14	11·906	57·2	40	54·906	104
4	6·097	39·2	15	12·699	59·0	50	91·982	122
5	6·534	41·0	16	13·635	60·8	60	148·791	140
6	6·998	42·8	17	14·421	62·6	70	233·093	158
7	7·492	44·6	18	15·357	64·4	80	354·643	176
8	8·017	46·4	19	16·346	66·2	90	525·450	194
9	8·574	48·2	20	17·391	68·0	100	760·000	212
10	9·165	50·0						

For intermediate temperatures, take the mean of the tensions at the temperatures above and below; the result, however, will not be absolutely correct, but approximate only—*e.g.*, find the tension at 60° F.; 60° F. = 15·5° C. Take the mean of the tensions at 15° C. and 16° C.

$$\begin{aligned} \text{Thus, } 15^\circ \text{ C.} &= 12{\cdot}699 \\ 16^\circ \text{ C.} &= 13{\cdot}635 \\ \hline & \quad 26{\cdot}334 \end{aligned}$$

$$\text{mean} = \frac{26{\cdot}334}{2} = 13{\cdot}167 = \text{tension at } 15{\cdot}5^\circ \text{ C., or } 60^\circ \text{ F.}$$

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